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ON THE WATER VAPOR IN THE ATMOSPHERE OVER THE UNITED STATES EAST OF THE ROCKY MOUNTAINS

By LOUIS P. HARRISON

[*Aerological Division, Weather Bureau, Washington, D. C.*]

CONTENTS

INTRODUCTION

I. PURPOSE OF INVESTIGATION

The purpose of this investigation is threefold:

1. To provide a practical method of computing the total mass of water vapor in the lower strata, i. e., to 3 or 4 kilometers, of the atmosphere based upon certain surface observations.
2. To deduce empirical equations based upon the mean values of available data for the lower strata for purposes of extrapolation to obtain tentative approximations of the mass of water vapor in the higher layers of the troposphere.
3. To ascertain and study the average distribution of water vapor in the lower strata of the atmosphere over the United States east of the Rocky Mountains.

II. THEORY OF METHOD

1. *General theory.*—From the gas laws, the mass of water vapor contained in a cubic meter of space is given by

$$1.060\,\frac{e_{mm.}}{1+\alpha t}=0.79507\,\frac{e_{mb.}}{1+\alpha t}=\text{absolute humidity, grams/cu. m.}$$

where e = vapor pressure in units indicated (mm. of mercury, or mb.).
t = temperature in °C.
α = thermal coefficient of cubical expansion, 0.00367.

If e_s = vapor pressure at the surface station, we may write for the absolute humidity at any height, h,

$$(1)\qquad W_h=Ke_s\,\frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}\text{ grams per cubic meter}$$

or

$$(1')\qquad W_h=Ke_s f_h\text{ grams per cubic meter}$$

where we define $f_h=\dfrac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}$, and where K has the value 1.060 when e_s is expressed in millimeters of mercury, and the value 0.79507 when e_s is expressed in millibars. The subscript h refers to the height at which the data are determined. The mass of water vapor in a layer of infinitesimal thickness dh and unit area is

$$(2)\qquad dS=W_h\,dh\text{ grams,}$$

whence S_a^b, the total mass of water vapor contained in a column of air 1 square meter in cross section and extending from $h=a$ to $h=b$ in meters above sea level, is

$$(3)\qquad S_a^b=\int_{h=a}^{h=b}W_h\,dh$$

Substituting equation 1 in equation 3 we get,

$$(4)\qquad S_a^b=Ke_s\int_{h=a}^{h=b}\frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}\,dh$$

or

$$(4')\qquad S_a^b=K\,e_s\,F_a^b\text{ grams,}$$

where by analogy we define $F_a^b=\displaystyle\int_a^b\frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}\,dh,$

the sub and super scripts referring to limits of integration.

From the empirical studies of Hann (1), Süring (2) and others, it has been shown that for average conditions the ratio $\left(\frac{e_h}{e_s}\right)$ is nearly constant for each height for widely differing geographical locations, and that it is independent of the value e_s. Hence we may express this value as a function of height,

$$(5)\qquad \left(\frac{e_h}{e_s}\right)=\theta(h).$$

Likewise with suitable restrictions upon place and time, for average conditions, we may express t_h as a function of height,

$$(6)\qquad t_h=\psi(h).$$

Hence it follows that with the proper restrictions, for average conditions, we find S to be a function of height, thus

$$(7)\qquad S_a^b=K\,e_s\int_a^b\frac{\theta(h)}{1+\alpha\psi(h)}dh.$$

It is clear that to determine the mass of water vapor in the given column of air of unit cross-section, we may either compute the value of the integral by numerical integration of equation 4, making use of empirical data, or we may obtain the functions $\theta(h)$ and $\psi(h)$ and integrate formally as indicated in equation 7.

2. *Application to the lower strata.*—From what has been stated above, in the case of numerical integration of equation 4 where empirical data are available, for a given place and season we should find the value of the integral F_a^b to be a constant for a given height of column $(b-a)$, under average conditions.

The evaluation of a sufficient number of such integrals for various places and seasons thus affords a simple means of computing the value S_a^b, provided that simple corrections to the values of the integrals may be found for places at heights above sea level different from those of the base stations, and provided also that geographic interpolations of the integrals are permissible. Under these circumstances the value e_s is determined currently and the value S_a^b thus computed is an approximation to the mass of water vapor in the given column of air. The actual value of this variable differs from the computed value depending upon the deviation of the current value of the integral F_a^b from its average value. Other factors which may introduce errors will be discussed in a later section (V).

The practicability of employing the alternative method of finding the value of the integral (i. e., determining the required functional relationships) depends to a great extent upon the complexity of the relationships and their variability with time and place. As may be seen from the data presented in the following section, the actual relationships differ in many small details both with respect to geographic location and to season. For practical purposes it is not essential to be able to reproduce the empirical values

$$f_h = \frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}$$

by means of an analytical function, if we have available empirical curves of this function plotted against height, or values of the areas under these curves for suitable limits. Therefore it has been decided to employ this method to determine the values of the integrals for the lower strata of the atmosphere where considerable observational data are available.

3. *Application to the higher strata.*—Thus far, at least three empirical equations have been deduced, giving the average value of the ratio $\left(\frac{e_h}{e_s}\right)$ as a function of height. The well-known equation of Hann (loc. cit.) based largely upon observations made at mountain stations gives

$$\left(\frac{e_h}{e_o}\right) = 10^{-\frac{h}{6300}}, \tag{8}$$

where h is the height in meters above sea level at which e_h is the vapor pressure, and e_o is the vapor pressure at sea level.

The equation deduced by Süring (loc. cit.) for the free air is

$$\left(\frac{e_h}{e_o}\right) = 10^{-\left(\frac{h}{6}+\frac{h^2}{120}\right)}, \tag{9}$$

where h is here expressed in kilometers.

Süring in the work previously mentioned, on testing the applicability of Hann's equation for values in the free air found that the use of one constant such as 6,500 gave values which were too great above 1 kilometer. However, by dividing up the height into several layers and using an appropriate constant for each layer, the data might be represented fairly closely by this equation. Thus it is stated in the Lehrbuch der Meteorologie of Hann and Süring (fourth edition, p. 244), that "For heights as high as 4.5 km., balloon observations show the constant to be 5,250 m. with good agreement; from 4.5 to 8 km. the constant is 3,550 m. on the average. (4,150 m. is found as the general average)."

On the basis of one year's observations at the Preussischen Aeronautischen Observatorium at Lindenberg, Hergesell (**3**) has found e_h as a function of temperature and therefrom, e_h as a function of height. He finds

$$\left(\frac{e_h}{e_s}\right) = 10^{10.231\left(\frac{t_h}{T_h}-\frac{t_s}{T_s}\right)} \tag{10}$$

where

t_h = temp. in °C. at height h.
t_s = temp. in °C. at the surface of the earth.
T_h = absolute temp. $(273+t)$ °K. at height h.
T_s = absolute temp. $(273+t)$ °K at surface.

Expressing $\left(\frac{t}{T}\right)$ as a function of height he finds for Lindenberg.

$$e_h = 7.046\times 10^{-\left(\frac{h}{8}+\frac{h^2}{48}\right)} \text{ mm. of mercury,} \tag{11}$$

where h = height above sea level in kilometers. Equation 10 showed good agreement with the means of observations at Batavia, except for values near the height 1.75 km. It was noted in this work that the data would have been fit more closely by the use of a third-order polynomial instead of one of the second order as shown.

Since the value $(1+\alpha t_h)$ does not differ very greatly from unity for temperatures in the troposphere, it is to be expected from the foregoing that only a first approximation to the function $f_h = \frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}$ is to be obtained by the use of an exponential function of the type given by Hann, and that closer approximation is obtained by the use of a higher polynomial in the expression. In this connection is may be noted that the evidence at hand shows quite conclusively that in general a Hann type equation gives values which are much too high at heights above 5 km. Thus in one set of data tried, such an equation gave values of the function at 10 km. equivalent to 200 per cent relative humidity.

Data based on a number of sounding balloon flights made in the United States showed for the interval 4–7 km. that the average variation of the function f_h with height could be represented fairly well by means of a second-order exponential function. A greater interval was not used since the hair hygrometer readings for greater heights were increasingly doubtful due to lag in the hygrometer elements (**4**).

Extrapolation of the function f_h in question by means of a second-order exponential expression is found to give reasonable values for high levels in the great majority of cases. The integration of the resulting function provides a means of obtaining the approximate mass of water vapor in the higher strata for which relatively few or no reliable observations are available.

III. THE EMPIRICAL DATA

The data used were obtained from the mean seasonal values of free-air vapor pressures and temperatures, for the stations shown in Table 1. In general, one observation was attempted each day.

TABLE 1.—*Sources of observations*

Station	Altitude, m. s. l.	Latitude N.	Longitude W.	Period of observations (inclusive) From—	To—	Length of record
	m.	° ′	° ′			*Yrs. Mos.*
Broken Arrow, Okla.	233	36 02	95 49	August, 1918	February, 1929	10 7
Drexel, Nebr.	396	41 20	96 16	November, 1915	March, 1926	10 5
Due West, S. C.	217	34 21	82 22	March, 1921	February, 1929	8 0
Ellendale, N. Dak.	444	45 59	98 34	January, 1918	February, 1929	11 2
Groesbeck, Tex.	141	31 30	96 28	October, 1918	February, 1929	10 5
Leesburg, Ga.	85	31 47	84 14	March, 1919	June, 1920	1 4
Naval Air Station, Washington, D. C.	7	38 54	77 03	July, 1925	February, 1929	3 8
Royal Center, Ind.	225	40 53	86 29	July, 1918	February, 1929	10 8

All of these stations with the exception of the naval air station at Washington, D. C., made the observations by means of kites and captive balloons. The latter station employed airplanes. Observations at the kite stations were usually begun between 7 and 8 a. m., local standard time, and generally lasted from 2 to 3½ hours. More or less variation in the time of beginning an observation was practised. In some cases launching of kites occurred before 7 a. m., and in others as late as 10 a. m. A small proportion of the flights were made during the afternoon. Airplane observations at Washington, D. C., during the period covered by the data showed no great regularity with regard to time of beginning. The flights in this case usually were started between 8 and 9 a. m., and lasted from 15 to 30 minutes. The data may thus be considered as representative of early to midmorning conditions.

The values of the function

$$\left\{\frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}\right\}$$

given in Table 2 were computed from corresponding seasonal means of vapor pressure and temperature, respectively. The seasonal means were computed from monthly means, each month's means being given equal weight. Each season was considered to be of three months duration, as follows:

Spring	March. April. May.	Autumn	September. October. November.
Summer	June. July. August.	Winter	December. January. February.

The method of differences was used in computing all means, i. e., the arithmetic mean of the surface values is first obtained, then the mean differences from level to level of daily or monthly observed values are computed and finally added successively to the surface mean to give the means for the various levels.

Table 2, which follows, also indicates the total number of daily observations upon which the computed values of the function are based. The seasonal mean *surface* vapor pressures and temperatures are tabulated in the first two columns.

TABLE 2.—*Seasonal values of the function* $f_h=\left\{\frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}\right\}$

BROKEN ARROW, OKLA. (Surface altitude 233 m., m. s. l.)

Season	Surface: Mean vapor pressure	Surface: Mean temperature	Designation (1)	Altitude above sea level, meters: Surface	250	500	750	1,000	1,250	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000	6,500	7,000
	Mb.	°*C.*																			
Spring	12.09	15.0	*a*	0.9478	0.9395	0.8339	0.7518	0.6877	C.6181	0.5498	0.4332	0.3437	0.2783	0.2296	0.1841	0.1479	*0.1300	0.1085	......	......	......
			b	778	778	778	774	768	740	695	586	455	330	192	90	40	14	6	......	......	......
Summer	23.21	26.1	*a*	0.9126	0.9050	0.8099	0.7324	0.6686	0.6077	0.5488	0.4421	0.3502	0.2808	0.2251	0.1786	0.1401	0.1121	*0.1082	0.1029	0.1002	......
			b	686	686	686	684	676	639	597	501	405	279	161	70	33	8	3	2	1	......
Autumn	13.60	16.6	*a*	0.9426	0.9360	0.8456	0.7689	0.7069	0.6445	0.5781	0.4476	0.3434	0.2664	0.2106	0.1536	0.1186	*0.0987	0.0910	......	......	......
			b	762	762	762	760	744	725	662	560	454	315	192	91	37	8	2	......	......	......
Winter	6.06	4.1	*a*	0.9852	0.9772	0.8738	0.7853	0.7105	0.6374	0.5729	0.4623	0.3848	0.3247	0.2771	0.2357	0.1934	0.1641	0.1392	*0.0962	......	......
			b	775	773	772	766	749	706	644	530	411	276	156	91	39	14	6	1	......	......

DREXEL, NEBR. (Surface altitude 396 m., m. s. l.)

Season	Surface: Mean vapor pressure	Surface: Mean temperature	Designation (1)	Altitude above sea level, meters: Surface	250	500	750	1,000	1,250	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000	6,500	7,000
Spring	8.22	9.3	*a*	0.9670	......	0.9190	0.8183	0.7439	0.6747	0.6099	0.4976	0.4120	0.3424	0.2802	0.2279	0.1873	0.1444	*0.1069	0.0819	0.0642	......
			b	903	......	903	901	876	845	801	693	547	415	242	110	42	20	10	6	2	......
Summer	18.87	22.9	*a*	0.9225	......	0.8724	0.7712	0.7023	0.6414	0.5807	0.4748	0.3848	0.3110	0.2506	0.1992	0.1653	0.1347	*0.1218	0.1134	......	......
			b	811	......	811	807	778	752	716	605	498	380	246	122	42	15	6	4	......	......
Autumn	9.52	11.1	*a*	0.9609	......	0.9211	0.8333	0.7639	0.6995	0.6396	0.5336	0.4443	0.3660	0.2977	0.2473	0.2054	0.1662	0.1349	*0.1025	0.0867	......
			b	867	......	867	863	849	828	793	700	589	473	330	171	72	24	13	5	2	......
Winter	3.66	−4.6	*a*	1.0172	......	0.9702	0.8885	0.8389	0.7959	0.7463	0.6417	0.5461	0.4609	0.3769	0.3002	0.2448	0.2067	*0.1826	......	......	......
			b	939	......	938	925	909	880	846	760	656	494	278	115	34	14	4	......	......	......

TABLE 2.—*Seasonal values of the function* $f_h=\left\{\dfrac{\left(\dfrac{e_h}{e_s}\right)}{1+\alpha t_h}\right\}$—Continued

Season	Surface: Mean vapor pressure	Surface: Mean temperature	Designation	Altitude above sea level, meters: Surface	250	500	750	1,000	1,250	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000	6,500	7,000
DUE WEST, S. C. (Surface altitude 217 m., m. s. l.)																					
	Mb.	°*C.*																			
Spring	12.13	16.4	a	0.9432	0.9286	0.8312	0.7591	0.7000	0.6422	0.5798	0.4561	0.3493	0.2655	0.2042	0.1636	0.1341	*0.1169	------	------	------	------
			b	524	524	524	512	481	438	403	334	253	166	104	51	16	9	------	------	------	------
Summer	22.30	26.3	a	0.9120	0.8997	0.8119	0.7464	0.6907	0.6353	0.5780	0.4731	0.3863	0.3135	0.2592	0.2102	0.1788	*0.1122	0.0632	------	------	------
			b	387	387	386	371	332	292	251	199	163	116	69	32	19	3	1	------	------	------
Autumn	13.81	17.0	a	0.9413	0.9272	0.8402	0.7721	0.7174	0.6570	0.5969	0.4735	0.3769	0.3109	0.2621	0.2232	0.2021	*0.1773	0.1670	------	------	------
			b	465	465	465	452	408	372	327	266	203	131	75	33	15	6	2	------	------	------
Winter	7.74	7.3	a	0.9739	0.9617	0.8763	0.8196	0.7644	0.7041	0.6390	0.5189	0.4080	0.3229	0.2561	0.2112	*0.1625	0.1386	------	------	------	------
			b	523	523	521	510	482	439	401	336	248	158	69	30	10	4	------	------	------	------
ELLENDALE, N. DAK. (Surface altitude 444 m., m. s. l.)																					
Spring	6.29	5.6	a	0.9798	------	0.9545	0.8446	0.7675	0.7039	0.6434	0.5308	0.4311	0.3443	0.2738	0.2135	0.1635	0.1279	*0.0952	0.0789	0.0655	0.0537
			b	949	------	948	945	931	901	851	730	580	415	252	130	56	20	6	3	2	1
Summer	15.85	20.0	a	0.9316	------	0.9049	0.7979	0.7216	0.6549	0.5903	0.4799	0.3957	0.3209	0.2631	0.2174	0.1802	0.1524	*0.1423	0.1326	------	------
			b	910	------	910	910	900	861	811	680	548	403	266	148	64	13	4	2	------	------
Autumn	7.51	6.4	a	0.9770	------	0.9579	0.8749	0.7995	0.7254	0.6578	0.5466	0.4571	0.3810	0.3101	0.2529	0.2043	0.1544	0.1224	0.1001	*0.0691	0.0640
			b	928	------	928	925	917	890	847	738	590	444	294	152	59	26	17	4	2	1
Winter	2.56	−10.1	a	1.0385	------	1.0178	0.9581	0.9219	0.8876	0.8354	0.7145	0.5984	0.4769	0.3656	0.2897	0.2461	0.1890	0.1435	*0.1354	------	------
			b	949	------	947	945	929	888	843	728	584	395	216	95	32	9	5	1	------	------
GROESBECK, TEX. (Surface altitude 141 m., m. s. l.)																					
Spring	15.43	17.9	a	0.9384	0.9010	0.8144	0.7375	0.6545	0.5683	0.4842	0.3589	0.2833	0.2328	0.1922	0.1619	0.1380	*0.1341	0.1206	------	------	------
			b	833	832	830	816	785	743	693	581	442	282	138	68	26	14	4	------	------	------
Summer	25.19	26.4	a	0.9117	0.8836	0.8082	0.7136	0.6206	0.5507	0.4923	0.3967	0.3238	0.2662	0.2191	0.1807	0.1463	*0.1311	------	------	------	------
			b	755	755	753	728	695	652	598	454	318	176	84	38	12	6	------	------	------	------
Autumn	17.21	18.8	a	0.9355	0.9032	0.8261	0.7533	0.6744	0.5989	0.5330	0.4099	0.3160	0.2416	0.1928	0.1503	0.1215	0.1013	0.0767	------	------	------
			b	761	761	756	732	696	643	592	494	396	278	154	84	35	13	4	------	------	------
Winter	9.26	9.0	a	0.9681	0.9290	0.8478	0.7772	0.6953	0.6228	0.5495	0.4300	0.3491	0.2842	0.2304	0.1964	0.1651	0.1424	0.1225	*0.1127	0.1068	0.1031
			b	844	844	840	808	762	708	650	552	426	299	163	79	43	21	6	1	1	1
LEESBURG, GA. (Surface altitude 85 m., m. s. l.)																					
Spring	14.98	20.0	a	0.9316	0.8611	0.7830	0.7214	0.6659	0.6100	0.5494	0.4066	0.3479	0.2820	*0.2549	0.2337	0.2200	------	------	------	------	------
			b	88	88	84	77	65	59	49	32	25	20	10	4	3	------	------	------	------	------
Summer	24.86	28.6	a	0.9050	0.8451	0.7830	0.7418	0.6855	0.6207	0.5526	0.4472	0.3677	0.3274	0.2848	*0.2598	------	------	------	------	------	------
			b	51	51	51	45	37	29	27	24	19	15	7	3	------	------	------	------	------	------
Autumn	18.40	23.5	a	0.9206	0.8702	0.8077	0.7453	0.6780	0.6177	0.5468	0.4065	0.2982	0.2282	0.1791	*0.1591	0.1298	------	------	------	------	------
			b	48	48	47	43	36	32	29	22	18	14	9	6	3	------	------	------	------	------
Winter	10.00	12.7	a	0.9555	0.8920	0.8102	0.7508	0.6856	0.6223	0.5612	0.4319	0.3541	0.2922	*0.2112	0.1545	0.1440	------	------	------	------	------
			b	67	67	66	60	56	53	48	44	37	25	10	4	2	------	------	------	------	------
NAVAL AIR STATION, WASHINGTON, D. C. (surface altitude 7 m., m. s. l.)																					
Spring	9.77	12.1	a	0.9575	0.8680	0.7752	0.6943	0.6376	0.5902	0.5495	0.4550	0.3632	0.2847	0.2322	0.1746	*0.1210	0.0790	0.0502	0.0286	------	------
			b	175	176	175	174	171	168	162	146	120	89	61	45	7	2	1	1	------	------
Summer	21.70	24.2	a	0.9184	0.8382	0.7491	0.6758	0.6143	0.5615	0.5195	0.4370	0.3448	0.2616	0.2001	0.1463	*0.0705	0.0374	0.0114	------	------	------
			b	175	175	175	171	169	165	161	144	127	106	38	30	2	1	1	------	------	------
Autumn	13.49	14.6	a	0.9491	0.8659	0.7909	0.7259	0.6738	0.6230	0.5706	0.4599	0.3553	0.2636	0.1936	*0.1325	0.0849	0.0420	------	------	------	------
			b	185	185	183	183	181	175	169	151	129	91	35	17	1	1	------	------	------	------
Winter	4.87	1.3	a	0.9953	0.9260	0.8523	0.7968	0.7393	0.6853	0.6327	0.5390	0.4545	0.3655	0.2944	*0.2459	0.2155	------	------	------	------	------
			b	149	149	146	146	144	142	139	124	103	60	19	12	2	------	------	------	------	------
ROYAL CENTER, IND. (surface altitude 225 m., m. s. l.)																					
Spring	8.84	10.2	a	0.9639	0.9492	0.8269	0.7437	0.6754	0.6111	0.5541	0.4536	0.3520	0.2775	0.2236	0.1814	0.1537	*0.1364	0.1273	0.1120	------	------
			b	704	704	704	693	665	633	597	486	359	242	136	73	39	18	7	1	------	------
Summer	18.69	23.4	a	0.9209	0.9090	0.8071	0.7415	0.6862	0.6271	0.5631	0.4382	0.3258	0.2479	0.1875	0.1456	*0.1238	0.1101	0.1051	0.0871	------	------
			b	588	584	583	564	529	491	460	388	286	182	104	44	16	4	3	1	------	------
Autumn	11.20	12.7	a	0.9555	0.9450	0.8479	0.7757	0.7043	0.6341	0.5669	0.4485	0.3529	0.2826	0.2265	0.1640	0.1345	*0.1109	0.0858	------	------	------
			b	722	720	719	701	662	618	583	485	392	274	138	54	24	7	2	------	------	------
Winter	4.32	−2.5	a	1.0093	0.9937	0.8834	0.8070	0.7321	0.6595	0.5992	0.4929	0.4200	0.3611	0.3016	0.2505	0.2162	0.1759	------	------	------	------
			b	773	773	773	752	729	680	634	528	370	208	96	31	7	1	------	------	------	------

[1] a=value of function f_h, b=number of observations.

* Values thus indicated and those for higher levels considered relatively doubtful.

Values of the function are computed to four decimal places; however, they are not to be regarded as accurate to that many figures, except possibly where based upon a large number of observations. In general, values based upon fewer than about 25 observations are considered to be in doubt in the second and possibly in the first decimal place. (See Secs. IV and V for discussions of errors.)

IV. COMPUTATION OF CONSTANTS OF THE EQUATIONS

1. *Graphical integration of equation (4) for given data.*—The function f_h has been plotted against height for the data given in TABLE 2. Some examples of the resulting curves are shown in Figures 1–8, for Ellendale, N. Dak., and Groesbeck, Tex., the most northern and southern stations, respectively, in the given group.

The evaluation of equation 4 was accomplished by drawing smooth curves through the plotted points as shown in the above figures, and reading the mean values of the ordinates f_h for each hundred-meter interval. The value of the definite integral is then obtained when the sum of the resulting mean ordinates is multiplied by 100. This method has advantages over the usual meth-

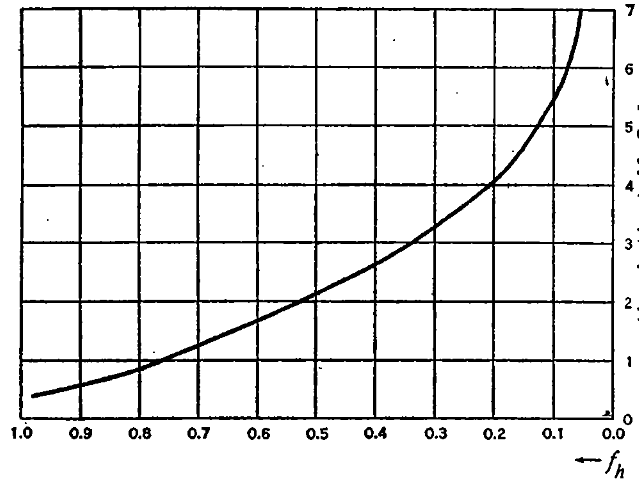

FIGURE 1.—Ellendale, N. Dak. Spring. f_h plotted against height

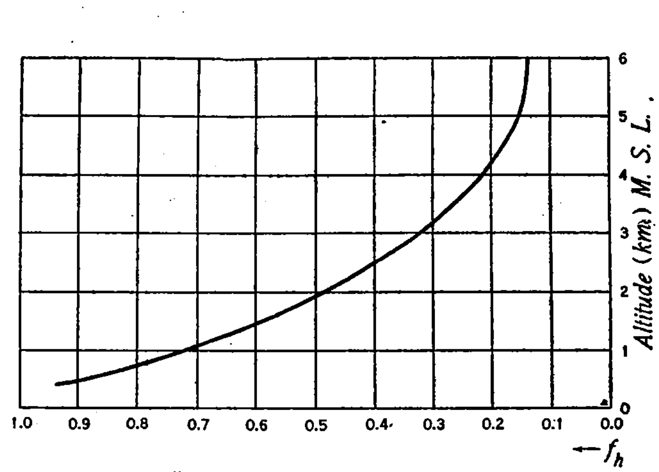

FIGURE 2.—Ellendale, N. Dak. Summer. f_h plotted against height

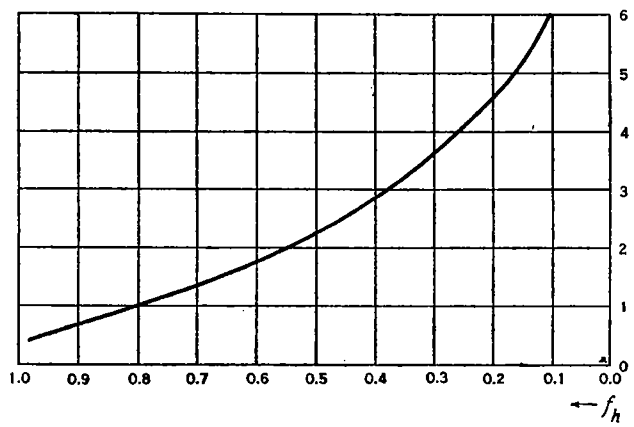

FIGURE 3.—Ellendale, N. Dak. Autumn. f_h plotted against height

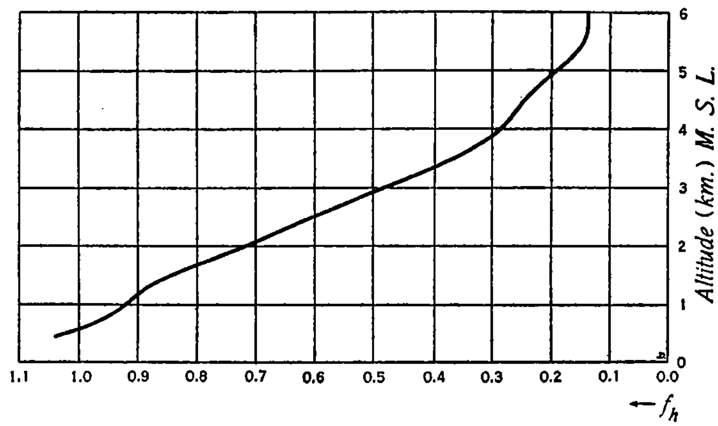

FIGURE 4.—Ellendale, N. Dak. Winter. f_h plotted against height

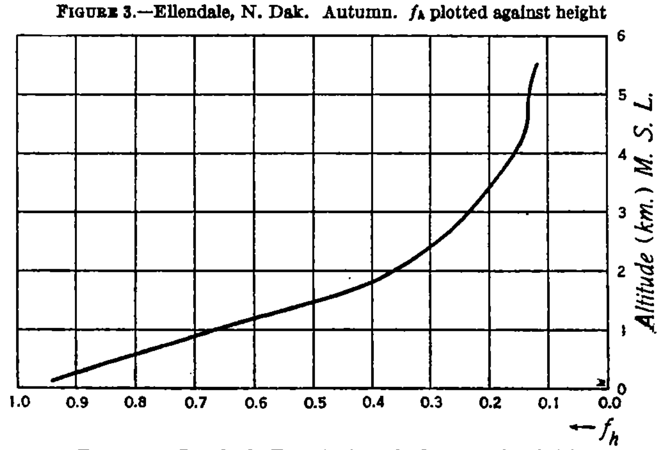

FIGURE 5.—Groesbeck, Tex. Spring. f_h plotted against height

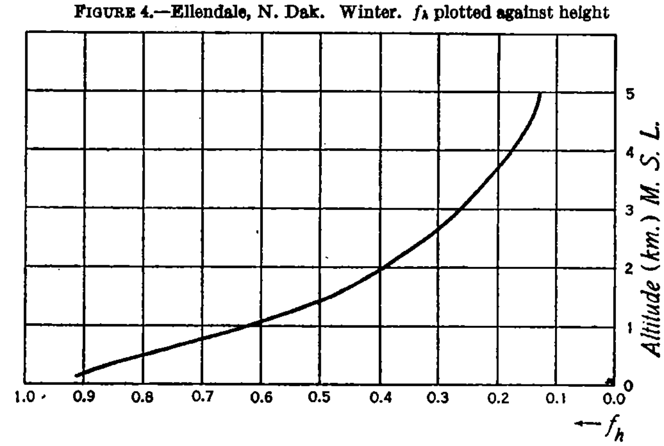

FIGURE 6.—Groesbeck, Tex. Summer. f_h plotted against height

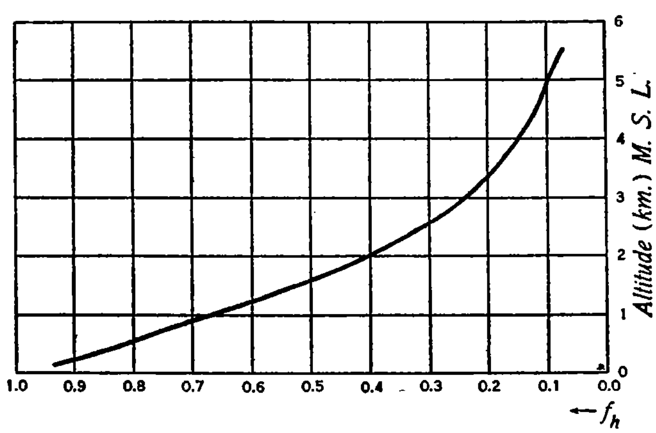

FIGURE 7.—Groesbeck, Tex. Autumn. f_h plotted against height

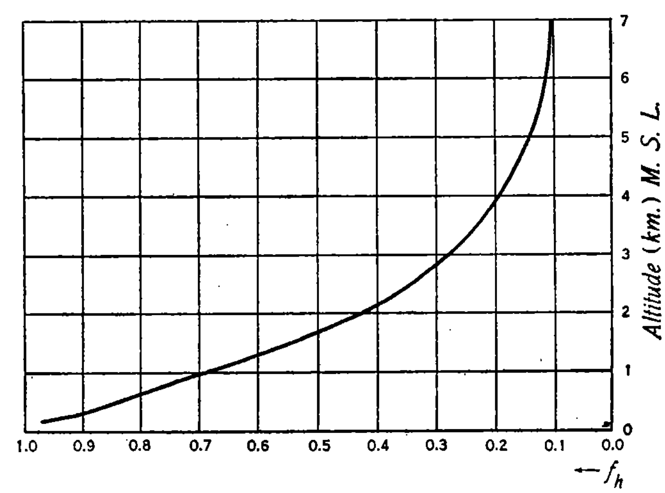

FIGURE 8.—Groesbeck, Tex. Winter. f_h plotted against height

ods of numerical integration such as Simpson's rules, since the curves could not be represented over their entire length by polynomials of any given degree. Adequate accuracy is attained by taking sufficiently narrow strips such as are indicated above (**5**).

Table 3 shows the results of the graphical integration of the functional values given in Table 2, and also, the corresponding values of the function S as defined in equation 4 for seasonal mean surface vapor pressures, where the surface heights above sea level are taken as the lower limits of integration and the indicated heights (h) are taken as the upper limits. We may thus compare both the values of F_s^h (defined in Table 3), which were required for use in equation 4, and the corresponding values of $\bar{S}_s^h$ for average conditions.

TABLE 3.—*Values of the integrals*[1]: $F_s^h=\int_s^h f_h dh=\int_s^h \frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}dh$ *and* $\bar{S}_s^h=K\bar{e}_s\int_s^h \frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}dh=K\bar{e}_s\cdot F_s^h$ *grams*

Upper limit-h	Spring		Summer		Autumn		Winter		Spring		Summer		Autumn		Winter	
	F_s^h	$\bar{S}_s^h$	F_s^h	$\bar{S}_s^h$	F_s^h	$\bar{S}_s^h$	F_s^h	$\bar{S}_s^h$	F_s^h	$\bar{S}_s^h$	F_s^h	$\bar{S}_s^h$	F_s^h	$\bar{S}_s^h$	F_s^h	$\bar{S}_s^h$
	BROKEN ARROW, OKLA. (233 m.)								GROESBECK, TEX. (141 m.)							
m.		*Kg.*		*Kg.*		*Kg.*		*Kg.*		*Kg.*		*Kg.*		*Kg.*		*Kg.*
500	236	2.27	228	4.21	237	2.56	247	1.19	314	3.85	309	6.19	316	4.32	324	2.39
1,000	615	5.91	595	10.98	622	6.72	641	3.09	682	8.37	664	13.30	691	9.45	711	5.23
1,500	924	8.88	899	16.59	944	10.20	959	4.62	966	11.85	939	18.81	991	13.56	1,020	7.51
2,000	1,169	11.24	1,144	21.11	1,196	12.93	1,216	5.86	1,172	14.38	1,159	23.21	1,225	16.76	1,262	9.29
2,500	1,361	13.08	1,341	24.74	1,393	15.06	1,427	6.88	1,333	16.36	1,338	26.80	1,406	19.23	1,455	10.71
3,000	1,516	14.57	1,498	27.64	1,545	16.70	1,604	7.73	1,461	17.93	1,485	29.74	1,544	21.12	1,611	11.86
3,500	1,642	15.78	1,624	29.96	1,664	17.99	1,754	8.45	1,567	19.23	1,605	32.15	1,652	22.60	1,739	12.80
4,000	1,744	16.76	1,726	31.84	1,754	18.96	1,881	9.06	1,655	20.31	1,704	34.13	1,737	23.76	1,846	13.59
4,500	1,827	17.56	1,805	33.30	1,822	19.70	1,989	9.58	1,730	21.23	1,785	35.75	1,804	24.68	1,936	14.25
5,000	*1,896	*18.22	1,868	34.46	*1,876	*20.28	2,078	10.01	*1,797	*22.05	*1,854	*37.14	1,860	25.44	2,013	14.82
5,500	1,956	18.80	*1,923	*35.48	1,923	20.79	2,154	10.38	1,861	22.83			1,906	26.07	2,079	15.31
6,000			1,976	36.46			*2,212	*10.66							*2,138	*15.74
6,500			2,027	37.40											2,193	16.14
7,000															2,245	16.53
	DREXEL, NEBR. (396 m.)								LEESBURG, GA. (85 m.)							
500	97	0.63	93	1.40	97	0.73	102	0.30	352	4.19	346	6.84	357	5.22	366	2.91
1,000	507	3.31	480	7.20	514	3.89	547	1.59	713	8.49	715	14.14	728	10.65	740	5.88
1,500	842	5.50	799	11.98	862	6.52	943	2.74	1,017	12.11	1,024	20.24	1,036	15.16	1,050	8.35
2,000	1,116	7.29	1,060	15.90	1,151	8.71	1,290	3.75	1,254	14.94	1,273	25.17	1,276	18.67	1,300	10.34
2,500	1,342	8.77	1,273	19.10	1,393	10.54	1,585	4.61	1,439	17.14	1,475	29.16	1,450	21.21	1,494	11.88
3,000	1,530	10.00	1,447	21.70	1,593	12.06	1,836	5.34	1,597	19.02	1,647	32.56	1,580	23.12	1,655	13.16
3,500	1,684	11.00	1,587	23.80	1,756	13.29	2,044	5.95	*1,731	*20.62	1,801	35.61	1,681	24.59	*1,782	*14.17
4,000	1,811	11.83	1,698	25.47	1,890	14.31	2,213	6.44	1,853	22.07	*1,935	*38.25	*1,766	*25.84	1,873	14.89
4,500	1,914	12.51	1,788	26.82	2,002	15.15	2,347	6.83	1,966	23.42			1,838	26.89	1,947	15.48
5,000	1,996	13.04	1,863	27.94	2,095	15.86	2,459	7.16								
5,500	*2,057	*13.44	*1,927	*28.90	2,170	16.42	*2,555	*7.44								
6,000	2,104	13.75	1,985	29.78	*2,228	*16.86										
6,500	2,140	13.98			2,276	17.23										
7,000																
	DUE WEST, S. C. (217 m.)								NAVAL AIR STA., WASHINGTON, D. C. (7 m.)							
500	248	2.39	242	4.29	250	2.74	261	1.61	426	3.31	412	7.11	426	4.57	456	1.77
1,000	628	6.06	614	10.89	636	6.98	669	4.12	776	6.03	751	12.95	789	8.47	853	3.30
1,500	948	9.14	931	16.51	965	10.60	1,018	6.26	1,070	8.31	1,034	17.84	1,101	11.81	1,194	4.62
2,000	1,205	11.62	1,191	21.12	1,228	13.48	1,306	8.04	1,319	10.25	1,270	21.91	1,357	14.56	1,484	5.75
2,500	1,405	13.55	1,402	24.86	1,439	15.80	1,537	9.46	1,522	11.82	1,465	25.27	1,558	16.72	1,731	6.70
3,000	1,557	15.02	1,574	27.91	1,609	17.67	1,719	10.58	1,683	13 07	1,616	27.88	1,710	18.35	1,935	7.49
3,500	1,674	16.14	1,716	30.42	1,751	19.23	1,863	11.46	1,812	14.08	1,732	29.88	1,823	19.56	2,097	8.12
4,000	1,766	17.03	1,833	32.50	1,872	20.55	1,979	12.18	1,914	14.87	1,817	31.34	*1,903	*20.42	*2,232	*8.64
4,500	1,839	17.74	1,930	34.22	1,977	21.71	*2,072	*12.75	*1,988	*15.44	*1,872	*32.29	1.957	21.00	2,346	9.08
5,000	*1,901	*18.33	*2,002	*35.50	*2,072	*22.75	2,146	13.21	2,037	15.82	1,897	32.72	1,988	21.33		
5,500			2,045	36.26	2,158	23.69			2,068	16.06	1,910	32.95				
6,000									2,088	16.22						
6,500																
7,000																
	ELLENDALE, N. DAK. (444 m.)								ROYAL CENTER, IND. (225 m.)							
500	54	0.27	51	0.64	54	0.32	58	0.12	242	1.70	235	3.49	245	2.18	258	0.89
1,000	478	2.39	453	5.71	494	2.95	539	1.10	614	4.32	606	9.01	631	5.62	660	2.27
1,500	830	4.15	779	9.82	857	5.12	981	2.00	919	6.46	918	13.64	947	8.43	990	3.40
2,000	1,122	5.61	1,044	13.15	1,157	6.91	1,369	2.79	1,171	8.23	1,168	17.36	1,199	10.68	1,260	4.33
2,500	1,361	6.81	1,262	15.90	1,407	8.40	1,697	3.45	1,371	9.64	1,357	20.17	1,398	12.45	1,488	5.11
3,000	1,555	7.78	1,440	18.14	1,616	9.65	1,966	4.00	1,526	10.72	1,499	22.28	1,556	13.86	1,682	5.78
3,500	1,709	8.55	1,585	19.97	1,788	10.68	2,174	4.42	1,650	11.60	1,607	23.88	1,683	14.99	1,848	6.35
4,000	1,830	9.15	1,704	21.47	1,929	11.52	2,334	4.75	1,750	12.30	1,689	25.10	1,782	15.87	1,986	6.82
4,500	1,923	9.62	1,804	22.73	2,042	12.19	2,467	5.02	1,833	12.88	*1,756	*26.09	1,855	16.52	2,102	7.22
5,000	1,995	9.98	1,887	23.78	2,132	12.73	2,576	5.24	*1,905	*13.39	1,814	26.96	*1,917	*17.07	2,200	7.56
5,500	*2,051	*10.26	*1,961	*24.71	2,202	13.15	2,658	5.41	1,971	13.85	1,867	27.74	1,966	17.51		
6,000	2,094	10.47	2,030	25.58	2,257	13.48	*2,727	*5.55	2,080	14.27	1,916	28.47				
6,500	2,131	10.66			*2,300	*13.73										
7,000	2,160	10.80			2,332	13.92										

[1] s=height of surface in meters above sea-level.
h=height of upper limit of column, meters above sea level.
$\bar{e}_s$=mean seasonal surface vapor pressure.
$\bar{S}_s^h$=mean (seasonal) mass of water vapor in column of air 1 sq. m. in cross-section and extending from surface to height h, in grams for K given as below (here given in *Kg.*).
K=1.060 when e_s is in mm.
K=0.79507 when e_s is in mb.
*Values thus indicated and those for higher levels considered relatively doubtful.

The values of F^h_s introduced above permit the computation of the mass of water vapor in a column of air from the ground to various heights above sea level, where the surface vapor pressure is known.

2. *Arbitrary selection of levels where values are considered relatively doubtful.*—As is evident from the curves shown (figs. 1–8), some irregularities exist in the data for the upper levels. Whether these irregularities are due to fewness of observations, instrumental errors, or represent a real average condition, it is impossible to say with certainty. Since some criteria are necessary to decide as to which values are sufficiently in error (relative to more reliable values for lower levels) to be discarded for present purposes, it was decided to use the following three indications as decisive in this matter:

(*a*) Number of observations,
(*b*) Smoothness of curves, f_h plotted against height, and
(*c*) Smoothness of curves, log f_h plotted against height.

The latter criterion is permissible since in general the function is exponential in nature.

In pursuance of this scheme all of the data were plotted upon semilogarithmic paper and curves drawn through the plotted points. Some examples of the resulting curves are shown in Figures 9 to 18 inclusive. Functions varying according to an equation similar to Hann's type, equation 8, appear here as straight lines, while those varying according to an equation similar to Süring's type, equation 9, appear as parabolas. Slight modifications of these two types are to be seen in Figures 9 and 10, respectively.

Finally the various curves were carefully examined and judged according to the criteria previously proposed. Levels at and above which the values f_h and F^h_s were considered relatively doubtful are indicated in Tables 2 and 3 by means of asterisks.

This procedure is of course rather arbitrary, but it is considered more desirable to weight the values in this manner than to present them as though having equal validity. It is the more important to do this since not all the curves can be reproduced.

Some of the values thus marked off are quite certainly less in doubt than others; however, no satisfactory absolute standard for comparison is known to exist. Values for Leesburg, Ga., and Washington, D. C., are considered to be much less reliable on the whole than values for the other stations, largely on account of the relative fewness of observations.

In addition, the effect of lag in the hair hygrometers used in the meteorographs is in general to make the indicated values too high, where the instrument goes from warmer to colder air (**4**). At low temperatures (below −30° C.) this effect has been found by Kleinschimdt (loc. cit.) to be quite large. The result of such an effect is to displace the logarithmic curves too far to the right. (See figs. 9–18.)

The curves for Washington, D. C., for all seasons, except winter, show a marked divergence in trend from the others in the high levels, indicating a rapid decrease of absolute humidity with height. This may be partly due to the fact that observations at the other stations were made during all kinds of weather except heavy or moderate rain or snow, whereas relatively fewer were made at the latter station on days when threatening, moist conditions prevailed at low levels. On the other hand, as has been noted by Gregg (**6**) and later by Wagner (**7**), temperatures in the free air are lower over the Atlantic coast than at corresponding latitudes in the interior of the continent. This is most strongly pronounced during the warmer seasons and at Northern stations. Hence we may draw the conclusion that the observed trend in the data for Washington, D. C., is probably indicative of the actual trend existing over that place. It may be noted that the few data available for heights above 4.5 km., for summer at Due West, S. C., indicate the same tendency.

3. *Tentative approximate computation of the constants F for the higher strata.*—A consideration of the factors governing the distribution of water vapor in the troposphere leads to the conclusion that the water vapor content above 4–5 km. should decrease more rapidly in geometric progression than below those heights. This is borne out by the smaller value of the constant found by Hann for the interval 4.5 −8 km. (See quotation from the Lehrbuch der Meteorologie previously given.) An examination of 91 sounding-balloon flights made in the United States showed that data for the interval 4–7 km. could be represented under average conditions by an equation of the form

$$f_h = f_d 10^{-[c_1(h-d)+c_2(h-d)^2]} \tag{12}$$

where

$$f_h = \text{value of the function } \frac{\left(\frac{e_h}{e_s}\right)}{1+\alpha t_h}$$

at height h, in meters.

f_d = known value of the same function at height d, the latter serving as a datum height, and c_1 and c_2 are constants.

The constant $-c_1$ represents the slope of the semilogarithmic curve at height d, h being taken as the independent variable.

The constant c_2 was found to have a seasonal and geographical variation. The data at hand did not give entirely consistent values of this constant, as was to be expected. The very approximate results thus obtained were smoothed out. Comparisons were then made to determine whether these results gave reasonable values of humidity at high elevations. Slight modifications were found necessary. The final tentative values are given in the following table:

TABLE 4.—*Tentative values of c_2**

Season	Northern stations	Southern stations
	$m.^{-2}$	$m.^{-2}$
Spring	2.6×10^{-8}	2.5×10^{-8}
Summer	2.0×10^{-8}	1.9×10^{-8}
Autumn	2.4×10^{-8}	2.1×10^{-8}
Winter	3.0×10^{-8}	2.7×10^{-8}

* The dimensions of c_2 are reciprocal square meters as indicated at the column heads

It may be noted that the constant 1/48 in Hergesell's equation (11) is equivalent to the constant 2.1×10^{-8} when h is expressed in meters.

Corresponding values of f_d and c_1 for the eight stations are given in Table 5.

The intervals of height from which the slopes $-c_1$ were obtained are also indicated. In general, the value f_d was chosen on the basis of the reliability criteria previously discussed.

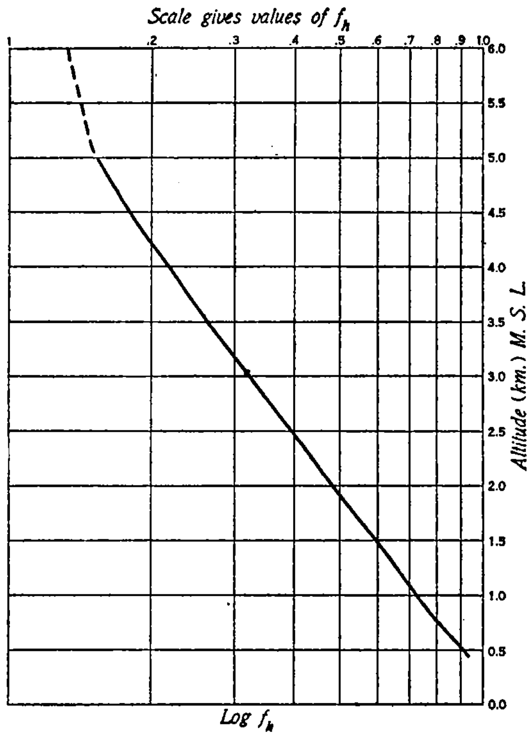

FIGURE 9.—Ellendale, N. Dak. Summer. $\log_{10} f_h$ plotted against height

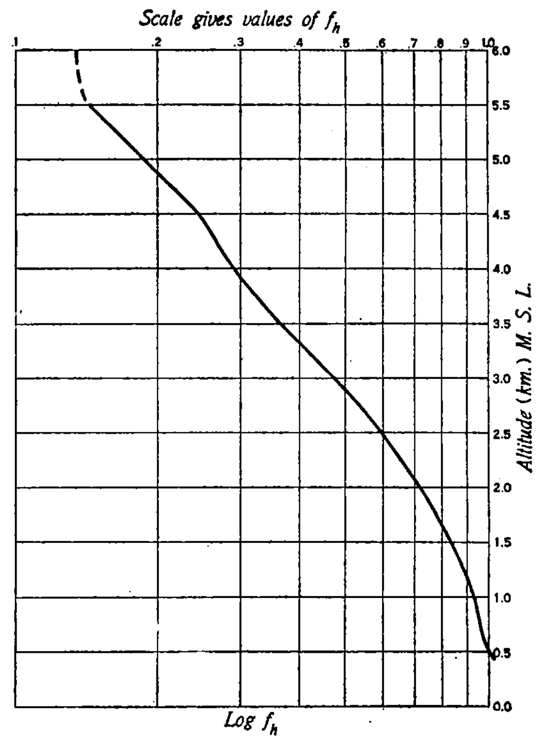

FIGURE 10.—Ellendale, N. Dak. Winter. $\log_{10} f_h$ plotted against height

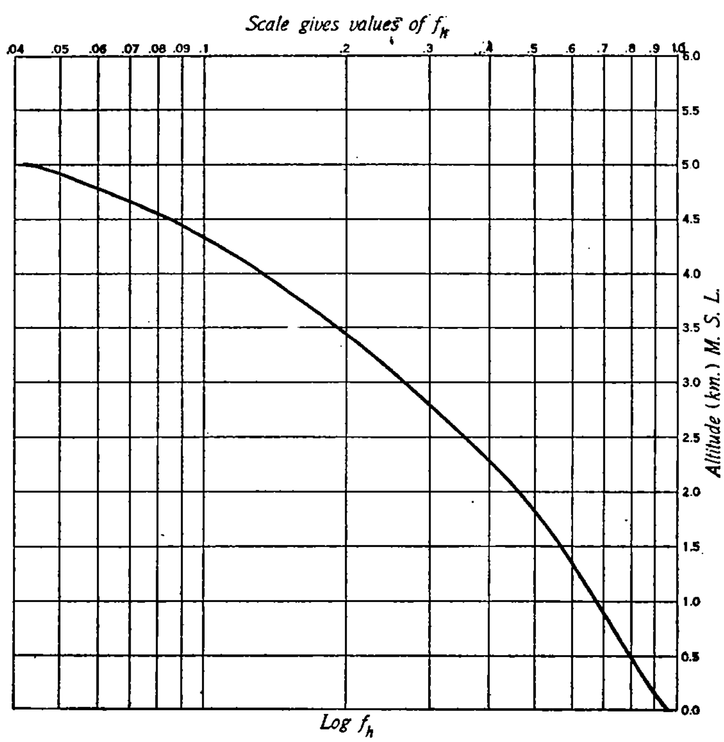

FIGURE 11.—Washington, D. C. Autumn. $\log_{10} f_h$ plotted against height

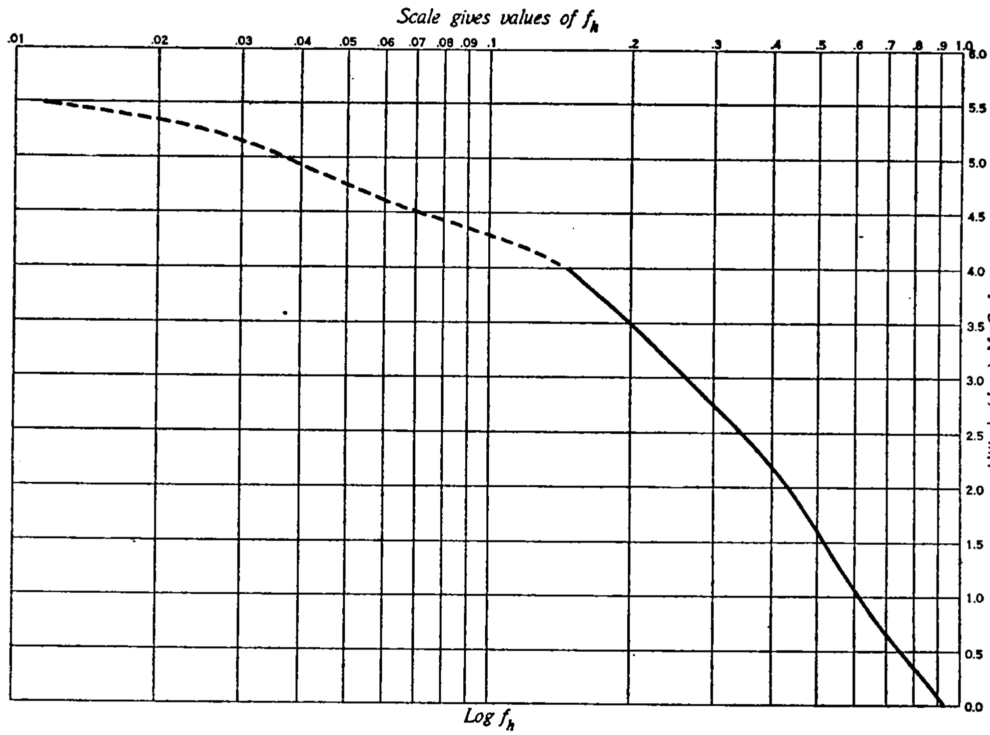

FIGURE 12.—Washington, D. C. Summer. $\mathrm{Log}_{10} f_h$ plotted against height

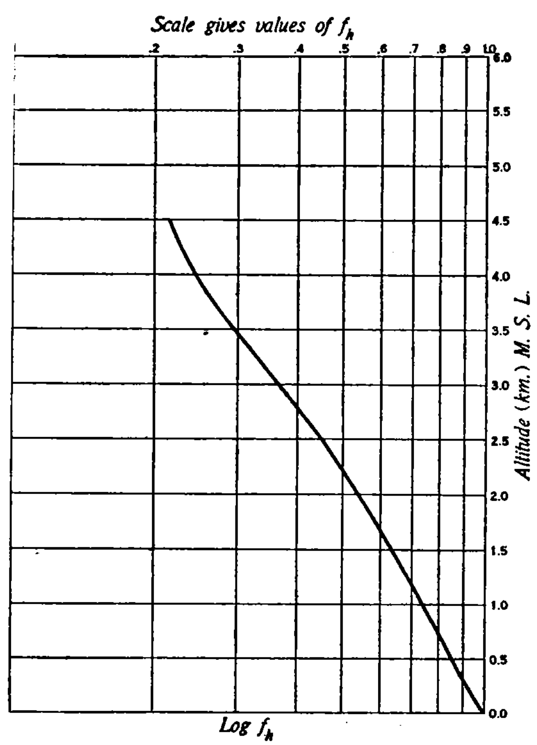

FIGURE 13.—Washington, D. C. Winter. $\mathrm{Log}_{10} f_h$ plotted against height

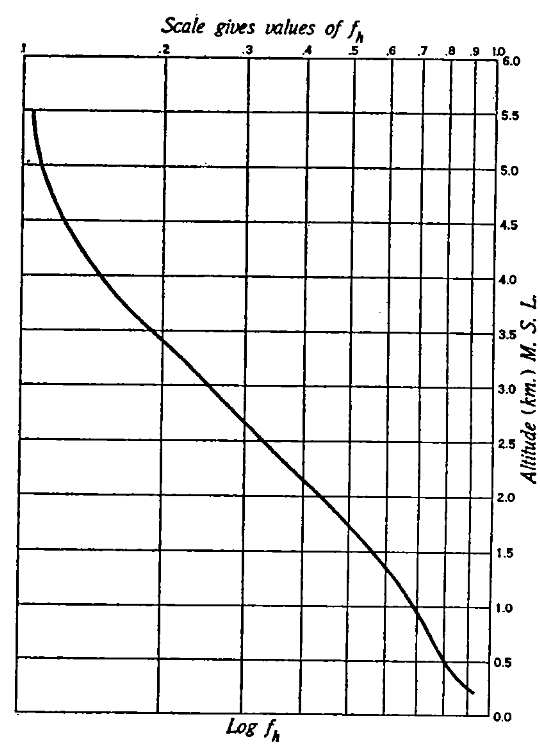

FIGURE 14.—Royal Center, Ind. Summer. $\mathrm{Log}_{10} f_h$ plotted against height

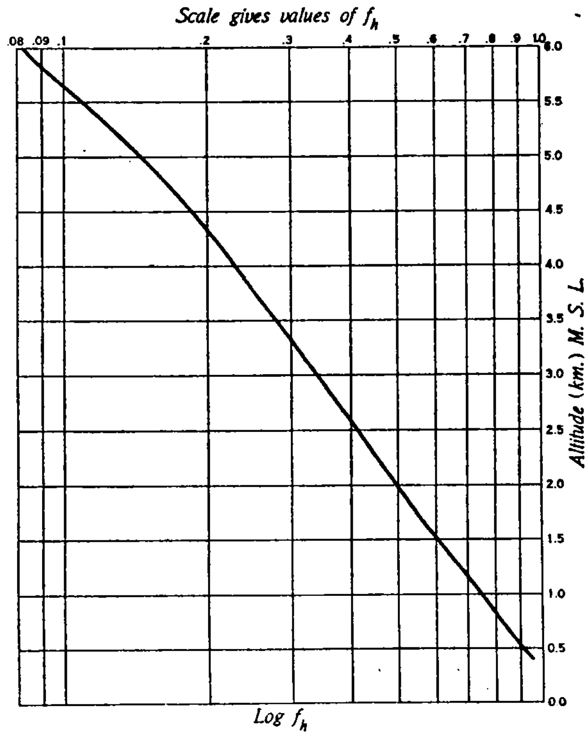

FIGURE 15.—Drexel, Nebr. Spring. $\log_{10} f_h$ plotted against height

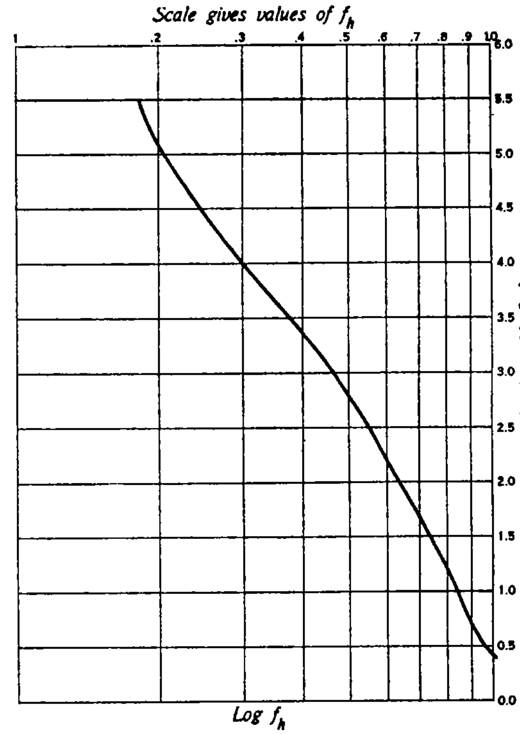

FIGURE 16.—Drexel, Nebr. Winter. $\log_{10} f_h$ plotted against height

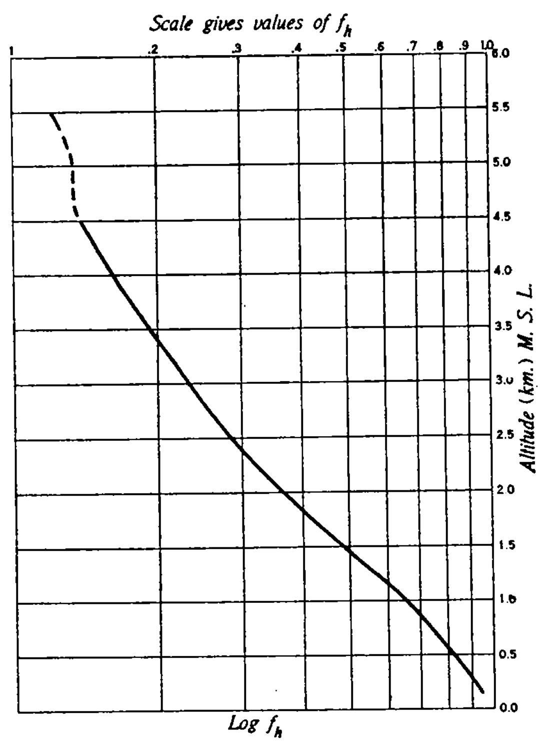

FIGURE 17.—Groesbeck, Tex. Spring. $\log_{10} f_h$ plotted against height

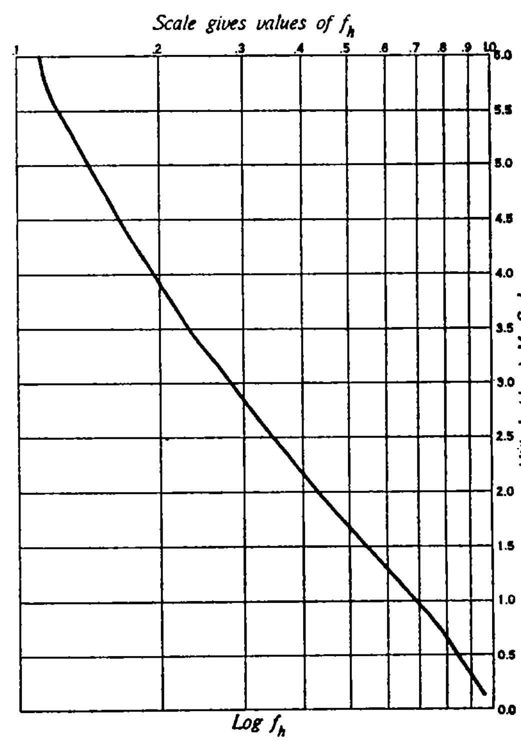

FIGURE 18.—Groesbeck, Tex. Winter. $\log_{10} f_h$ plotted against height

TABLE 5.—*Data used for extrapolation of f_h to heights greater than for which it is available, according to equation 12*

Station	Spring				Summer			
	d	f_d	c_1	Interval [1]	d	f_d	c_1	Interval [1]
	m.		10^{-4} *m.*$^{-1}$	*Km.*	*m.*		10^{-4} *m.*$^{-1}$	*Km.*
Broken Arrow, Okla	4,500	0.1479	1.90	4.0-4.5	4,500	0.1401	2.11	4.0-4.5
Drexel, Nebr	5,000	.1444	2.26	4.5-5.0	5,000	.1347	1,78	4.5-5.0
Due West, S. C	4,000	.1636	1.93	3.5-4.0	4,000	.2102	1.82	3.5-4.0
Ellendale, N. Dak	5,000	.1279	2.13	4.5-5.0	4,500	.1802	1.63	4.0-4.5
Groesbeck, Tex	4,000	.1619	1.49	3.5-4.0	4,500	.1463	1.75	3.5-4.5
Leesburg, Ga	3,000	.2820	1.82	2.5-3.0	2,500	.3677	1.70	2.0-2.5
Naval Air Station, Washington, D. C	3,500	.2322	2.12	3.0-4.0	3,500	.2001	2.52	3.0-4.0
Royal Center, Ind	4,000	.1814	1.82	3.5-4.0	4,000	.1456	1.80	3.5-4.5

Station	Autumn				Winter			
	d	f_d	c_1	Interval [1]	d	f_d	c_1	Interval [1]
	m.		10^{-4} *m.*$^{-1}$	*Km.*	*m.*		10^{-4} *m.*$^{-1}$	*Km.*
Broken Arrow, Okla	4,500	0.1186	2.25	4.0-4.5	4,500	0.1984	1.72	4.0-4.5
Drexel, Nebr	6,000	.1025	2.39	5.5-6.0	4,500	.2448	1.77	4.0-4.5
Due West, S. C	3,500	.2621	1.48	3.0-3.5	4,000	.2112	1.67	3.5-4.0
Ellendale, N. Dak	5,000	.1544	2.22	4.5-5.0	5,500	.1435	2.34	4.5-5.5
Groesbeck, Tex	4,500	.1215	1.85	4.0-5.0	4,500	.1651	1.51	4.0-4.5
Leesburg, Ga	3,500	.1791	1.57	3.0-4.0	3,000	.2922	1.67	2.5-3.0
Naval Air Station, Washington, D. C	3,000	.2636	2.59	2.5-3.0	3,500	.2944	1.88	3.0-3.5
Royal Center, Ind	4,500	.1345	1.72	4.0-4.5	4,000	.2505	1.61	3.5-4.0

[1] Columns thus headed indicate interval of data from which value c_1 was obtained.

Examples of results of computation by means of equation 12 are shown in Table 6 for autumn at Groesbeck, Tex. Mean temperatures were obtained by applying the mean lapse rates obtained from the sounding-balloon series of October, 1927, made at that station (8) to the mean temperature at 4 km., which had been obtained from kite records for the season in question. Vapor pressures were computed by using the temperatures found as described above to give $e_h = f_h[\bar{e}_s(1+at_h)]$. Relative humidities were computed by dividing the computed vapor pressures e_h by the saturated vapor pressures corresponding to the given temperatures.

TABLE 6.—*Examples of use of equation 12*

Groesbeck, Tex.—Autumn season				
h	t	f_h	e_h	Relative humidity
COMPUTED VALUES				
Km.	°*C.*		*mb.*	%
4.5	−1.4	0.1215	2.08	38
5.0	−4.3	.0971	1.64	38
5.5	−7.6	.0756	1.26	39
6.0	−11.0	.0575	.949	40
7.0	−18.3	.0310	.498	41
8.0	−26.2	.0152	.236	42
9.0	−33.7	.00671	.101	39
10.0	−40.5	.00270	.0396	33
11.0	−46.4	.000988	.0141	24
12.0	−51.4	.000328	.00458	14
13.0	−55.7	.0000988	.00135	7
14.0	−59.8	.0000270	.000363	3
15.0	−63.6	.0000067	.000089	1
OBSERVED VALUES (KITES)				
4.5	−1.8	0.1215	2.08	[1] 37
5.0	−3.8	.1013	1.72	35
5.5	−6.2	.0767	1.29	31

[1] Based on 35, 13, and 4 observations, respectively. Latter appears too low.

It is to be noted that computation makes the relative humidity a maximum near 8 km., which is the region of maximum average lapse rates found in the troposphere.

Similar comparisons for the other stations show that the great majority give reasonable values of humidities, a few giving some values which seem somewhat too large and a few giving values which seem too small. Drexel, Nebr., for autumn was among the former, and Washington, D. C., among the latter.

It is now necessary to integrate equation 12. This may be done by numerical integration, or more formally by expressing the function in an infinite series which is uniformly convergent and which hence may be integrated term by term. Still another method is to integrate by parts and express the resulting integral in terms of an infinite series by a process of continued integration (**9**). These methods are necessarily laborious. However, by making suitable transformations as shown in the following, the definite integral may be quickly computed from tables already in existence. To do this, equation 12 is converted to the more convenient exponential form

$$f_h = f_d e^{-[c_2\kappa(h-d)^2 + c_1\kappa(h-d)]} \tag{13}$$

where e = base of Napierian logarithms
and $\kappa = \log_e 10 = 2.3026-$ = reciprocal of the modulus of common logarithms.

Completing the square in the exponent we get

$$f_h = f_d e^{\frac{c_1^2\kappa}{4c_2}} e^{-\left[\sqrt{c_2\kappa}\,(h-d) + \frac{c_1\sqrt{\kappa}}{2\sqrt{c_2}}\right]^2} \tag{14}$$

This reduces to

$$f_h = f_d 10^{\frac{c_1^2}{4c_2}} e^{-[\sqrt{c_2\kappa}]^2\left[h + \left(\frac{c_1}{2c_2} - d\right)\right]^2} \tag{15}$$

Letting

$$N = f_d 10^{\frac{c_1^2}{4c_2}}$$

$$a = \sqrt{c_2\kappa}$$

$$b = \left(\frac{c_1}{2c_2} - d\right),$$

the last equation simplifies to the form

$$f_h = N e^{-a^2[h+b]^2} \tag{16}$$

The desired integral is

$$\int_d^H f_h dh = N\int_d^H e^{-a^2[h+b]^2} dh \tag{17}$$

where H is the upper limit of integration.

From the geometry of the respective curves it becomes evident that

$$N\int_0^{h_1} e^{-a^2(h+b)^2} dh = N\int_0^{h_1+b} e^{-a^2h^2} dh - N\int_0^b e^{-a^2h^2} dh \tag{18}$$

where h_1 is any upper limit of integration. But since obviously

$$N\int_d^H e^{-a^2(h+b)^2} dh = N\int_0^H e^{-a^2(h+b)^2} dh - N\int_0^d e^{-a^2(h+b)^2} dh, \tag{19}$$

on substituting equation 18 into the right-hand members of equation 19, respectively, we get

$$(20)\qquad N\int_d^H e^{-a^2(h+b)^2}\,dh = N\int_o^{H+b} e^{-a^2h^2}\,dh - N\int_o^{d+b} e^{-a^2h^2}\,dh.$$

Let $t = ah$
then $dt = a\ dh$
and $dh = \frac{dt}{a}$, whence we have

$$(21)\qquad N\int_o^h e^{-a^2h^2}dh = \frac{N}{a}\int_o^{ah=t} e^{-t^2}dt.$$

Substituting equation 21 in equation 20 we obtain

$$(22)\qquad N\int_d^H e^{-a^2(h+b)^2}dh = \frac{N}{a}\int_o^{a(H+b)} e^{-t^2}dt - \frac{N}{a}\int_o^{a(d+b)} e^{-t^2}dt.$$

There are numerous tables available of the definite integral (**10**):

$$\theta(t) = \frac{2}{\sqrt{\pi}}\int_o^t e^{-t^2}dt$$

much used in the Theory of Probability. To adapt equation 22 for the use of such tables we rewrite it in the form

$$(23)\qquad N\int_d^H e^{-a^2(h+b)^2}dh = \left(\frac{\sqrt{\pi}}{2a}\right)N\left\{\frac{2}{\sqrt{\pi}}\int_o^{a(H+b)} e^{-t^2}dt - \frac{2}{\sqrt{\pi}}\int_o^{a(d+b)} e^{-t^2}dt\right\}$$

Noting that $a(d+b) = a\left(\frac{c_1}{2c_2}\right)$, we have finally for the special case where $H = \infty$ and $a \neq 0$, that

$$(24)\qquad F_d^\infty = N\int_d^\infty e^{-a^2(h+b)^2}dh = \left(\frac{\sqrt{\pi}}{2a}\right)N\left\{1 - \frac{2}{\sqrt{\pi}}\int_o^{a\left(\frac{c_1}{2c_2}\right)} e^{-t^2}dt\right\}$$

where

$N = f_d 10^{\frac{c_1^2}{4c_2}}$
$a = \sqrt{c_2\kappa}$
$\kappa = 2.3026-$, the other values as shown in Tables 4 and 5.

Table 7 (column F_d^∞) shows the results of integration for the higher strata, according to equation 24. Taking the upper limit as infinity introduces no significant error. The corresponding integrals for the lower strata are also given as well as the sums of the two respective integrals.

TABLE 7.—*Values of the factors F applying from the surface to the limits of the atmosphere*

Station	Spring				Summer			
	d	F_s^d	F_d^∞	[1] F_s^∞	d	F_s^d	F_d^∞	[1] F_s^∞
	Km.				*Km.*			
Broken Arrow, Okla	4.5	1,827	247	2,074	4.5	1,805	229	2,034
Drexel, Nebr	5.0	1,996	215	2,211	5.0	1,863	245	2,108
Due West, S. C	4.0	1,766	271	2,037	4.0	1,833	380	2,213
Ellendale, N. Dak	5.0	1,995	198	2,193	4.5	1,804	346	2,150
Groesbeck, Tex	4.0	1,655	311	1,966	4.5	1,785	272	2,057
Leesburg, Ga	3.0	1,597	485	2,082	2.5	1,475	694	2,169
Naval Air Station, Washington, D. C	3.5	1,812	363	2,175	3.5	1,732	289	2,021
Royal Center, Ind	4.0	1,750	309	2,059	4.0	1,689	263	1,952

Station	Autumn				Winter			
	d	F_s^d	F_d^∞	[1] F_s^∞	d	F_s^d	F_d^∞	[1] F_s^∞
	Km.				*Km.*			
Broken Arrow, Okla	4.5	1,822	183	2,005	4.5	1,989	338	2,327
Drexel, Nebr	6.0	2,228	149	2,377	4.5	2,347	411	2,758
Due West, S. C	3.5	1,751	526	2,277	4.0	1,979	375	2,354
Ellendale, N. Dak	5.0	2,132	236	2,368	5.5	2,658	204	2,862
Groesbeck, Tex	4.5	1,804	213	2,017	4.5	1,936	309	2,245
Leesburg, Ga	3.5	1,681	348	2,029	3.0	1,655	519	2,174
Naval Air Station, Washington, D. C	3.0	1,710	369	2,079	3.5	2,097	488	2,585
Royal Center, Ind	4.5	1,855	241	2,096	4.0	1,986	443	2,429

[1] See equation 4″ following, and text immediately thereafter.

We may note that F_s^∞ according to its definition by equation 4′, or

$$(4'')\qquad S_s^\infty = K\ e_s\ F_s^\infty \text{ grams},$$

provides a means of computing approximately the mass of precipitable water vapor in a column one square meter in cross section and extending from the ground to the limits of the atmosphere. The function F_h^∞ is independent of the units in which the surface vapor pressure, e_s, is expressed. The value K, however, for our purposes, depends only upon the units in question. For convenience, we note here that

$K = 1.060$ when e_s is in mm. mercury.
$K = 0.79507$ when e_s is in mb.
$K = 26.92$ when e_s is in inches of mercury.

It may be reiterated that the term F_d^∞ is only tentative. More reliable results can only be obtained by means of direct spectroscopic observations (**11**) to determine the desired values, or at least in part by means of reliable aerological observations, particularly of humidity, to great heights.

To obtain the desired value S_x^h for a station at height x differing from the height of the nearest of the 8 stations given herein, the surface vapor pressure e_x may be reduced to the corresponding vapor pressure at the surface of the "datum station," e_s, by the use of Hann's equation for mountain stations, thus

$$(8')\qquad e_s = e_x\ 10^{\frac{(x-s)}{6300}}$$

In addition, the factor F_s^h obtained from Table 3 or 7 must be reduced by the amount F_s^x obtained from Table 3. Consequently, the final corrected value is

$$(25)\qquad S_x^h = K\ e_x 10^{\frac{(x-s)}{6300}}\ (F_s^h - F_s^x) \text{ grams}.$$

V. DISCUSSION OF FORMULAS; SOURCES OF ERRORS

1. *Comparisons with other formulas.*—Hann (**12**) has found that by changing the constant of his equation, 8, to make it conform more closely to conditions in the free air (i. e. changing from 6300 to 5000) and neglecting the temperature factor $(1 + a\ t_h)$, he gets what is equivalent to the expression,

$$(26)\qquad S_o^\infty = K\ e_o\ (2170) \text{ grams}.$$

The value in parenthesis compares closely with the average of the corresponding factors given in Table 7. Humphreys (**13**) has found from 74 balloon observations made in Europe that the yearly average for *clear* days is closely representable by what is equivalent to the expression

$$(27)\qquad S^\infty = K\ e_s\ (1930) \text{ grams},$$

approximately, where h_s averaged between 200 and 300 meters. Here the agreement is reasonably close with the values for the warmer seasons—i. e., seasons with minimum cloudiness.

Fowle's spectro-bolometric observations on Mount Wilson (**11**) showed the mean value of F to be approximately half way between Hann's and Humphreys' values, or $F_o^\infty = 2040$ nearly, using Hann's equations for reduction to sea level. This value is based on observations made during the months June–September, inclusive, 1910 and 1911.

2. *Sources of error in the formulas and results.*—As may readily be seen from the foregoing, the original assumptions that the ratio $\left(\frac{e_h}{e_s}\right)$ and t, or f_h, are explicit functions of height reduce to the proposition that the amount of water vapor over any small area of earth's surface is directly proportional to the vapor pressure at the surface. This is equivalent to saying that F^{∞}_{s} is a constant independent of factors other than the height s. This is of course untrue, for obviously the value in question varies with time and with changing meteorological conditions in the atmosphere.

Where the time limit is sufficiently extended, the relationships may be expected to hold quite closely provided that unusual meteorological deviations from the average have not occurred. The relationship is also valid at times when a close approximation to the statistical "average condition" prevails.

(*a*) *Checking of normal exchange.*—The apparent constancy of the ratio $\left(\frac{e_h}{e_s}\right)$ found under the circumstances described has its foundation in the combined operations of convection, and mixing and diffusion of water vapor in the lower atmosphere. When little convection and mixing are occurring from the ground upward as may be the case where a strong inversion exists not far above ground, the average law of variation of this ratio with height may be departed from considerably. The ground may thus heat up, causing increased evaporation and thus increased vapor pressure, while almost no exchange is taking place between the ground layer and the layers above the inversion. The conditions above the inversion may consequently be largely tempered by the winds at those levels and regions from which the winds are blowing.

The relation which obtains between aqueous vapor at two levels in a convecting mass of air in which condensation and mixing has not yet taken place may be expressed simply by the equation

$$\frac{e_2}{p_2} = \frac{e_1}{p_1} \tag{28}$$

where e_1, p_1 are the vapor pressure and barometric pressure respectively at the original level, and e_2, p_2 are the corresponding values at a subsequent level. As an example of the average distribution of vapor pressure in the lower layers of the troposphere, we may cite the empirical equations found for average values during the spring season at Drexel, Nebr.,

$$\frac{e_h}{p_h} = \frac{e_s}{p_s} 10^{-c_3(h-s)} \tag{29}$$

which applies from the surface $h \equiv s = 396$ m. to $h = 750$ m. (above sea level) and,

$$\frac{e_h}{p_h} = \frac{e_d}{p_d} 10^{-c_4(h-d)} \tag{30}$$

which applies from $h \equiv d = 750$ m. to $h = 3500$ m., c_3 and c_4 being constants.

From the data at hand we find

$c_3 = 1.625 \times 10^{-4}$ (for h in meters)

$c_4 = 1.231 \times 10^{-4}$ (where $d = 750$ m.)

and $\frac{p_d}{p_s} = 0.958$.

These relationships show that, statistically, convection, turbulence and diffusion with the resultant mixing and condensation cause the ratios $\left(\frac{e}{p}\right)$ not to remain constant with height but to decrease in geometric ratio with increasing height.

It may be noted that in this case since $c_3 > c_4$, the ratio in question decreases more rapidly from the ground (396 m.) to the height 750 m. above sea level than it does from 750 m. to 3,500 m. The effect of temperature lapse rates may now be seen from the values given in Table 8 following.

TABLE 8.—*Mean spring lapse rates, Drexel, Nebr.*

Interval	$-\frac{\Delta t}{\Delta h}$, °C./100m.	Interval	$-\frac{\Delta t}{\Delta h}$, °C./100m.
m.		*m.*	
396–500	0.67	1,500–2,000	0.44
500–750	.60	2,000–2,500	.52
750–1,000	.40	2,500–3,000	.56
1,000–1,250	.36	3,000–3,500	.58
1,250–1,500	.36	3,500–4,000	.58

It is evident from these values that convection is here relatively stronger in the first 350 m. above ground than above that height. The small lapse rates from 750–2,000 m. are due statistically to the inversions prevalent over northern stations during winter and early spring (**14**). Thus, as the generally moist ground warms up in spring, convection and turbulence raise considerable water vapor from the layers adjacent thereto, carrying it up to the region of small or inverted lapse rates where the convection is checked. From there the water vapor, tends to slowly diffuse upward, aided somewhat by the higher (gradient) wind velocities occurring at those levels, but since lapse rates in these layers are below adiabatic, eddy diffusion carries a portion of the water vapor back toward the ground layers. In addition, since the ground is comparatively moist in this season due to the after effects of the winter frost and snow cover, evaporation proceeds very rapidly near the ground especially during clear days, often adding water vapor to the ground layers more quickly than it can be carried aloft. This explains why the ratio $\left(\frac{e}{p}\right)$ decreases more slowly in the layer from 750–2,000 m., than it does immediately below it.

The concept under consideration is perhaps further verified by comparing the variation of these ratios with height for winter and summer at Ellendale, N. Dak.

Figure 19 shows plots of $\left[h, \log_{10}\frac{e}{p}\right]$ for the two seasons in question. The Summer curve is perhaps typical of average conditions when the stirring processes of the atmosphere have full play. The Winter curve shows the influence of the inversion in the lower layers. The mean seasonal lapse rates are shown by the small figures adjacent to each interval of height. The inversions in question are largely the result of the frequent "anticyclonic weather with its clear skies and intense radiation" (**6**) observed in these regions. The strong cooling of the lower layers due to radiation after sunset produces a subsidence of the air which thus becomes dynamically warmed. The continued cooling of the ground finally causes the temperature of the air at that level to become lower than that of the free air immediately above. The water vapor brought down by the subsidence of air thus finds itself in a region of diminished lapse rate and finally in an inversion. Convection is effectively checked under such circumstances and the relative proportions of the constituent gases of the atmosphere tend to become fixed in amount. The evaporation of liquid or solid water falling through the inversion provides an important source of water vapor for the inversion layer when precipitation occurs. The water vapor, being less dense than dry air tends to diffuse molecularly toward the top of the inversion. Eddy diffusion, however, under the influence of increased wind velocities in the inversion layer plays an opposing rôle in the mechanism of the process, aiding in the general mixing of this constituent largely in the downward direction. The facts just adduced explain in part why the curve for winter is nearly vertical from the ground to about 1,000 m. elevation above.

Since molecular diffusion in the absence of convection and turbulence is relatively slow as an agency for dissi-

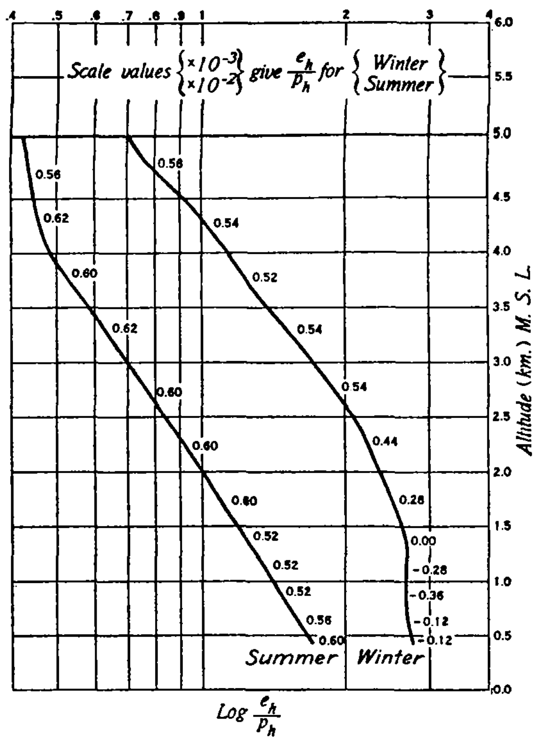

FIGURE 19.—Ellendale, N. Dak. Summer and winter. $\text{Log}_{10}\left(\frac{e}{p}\right)$ plotted against height. Small figures adjacent to curves are mean lapse rates for interval in °C./100 m. The winter values are less in absolute magnitude than the summer values. Winter surface value, $\left(\frac{e_s}{p_s}\right)=0.00275$; summer surface value, $\left(\frac{e_s}{p_s}\right)=0.0170$

pating water vapor, under the conditions outlined above, changes in the surface vapor pressure, say, due to surface heating at sunrise, are bound to take some time in making themselves felt at higher levels. It may also be noted that the higher temperatures in the inversion increase the capacity of the space for water vapor so that relatively large amounts of vapor may be present without condensing.

Thus, cases of abnormally large factors F_s^∞ observed by Fowle at Mount Wilson (**11**), height 1,730 m., may be due to the forced convection of a stratum of air over the mountain, the air being of oceanic origin and having a strong inversion and low humidity at the height in question. Such conditions are very common in the Summer on the California coast (**15**). Thus with near normal moisture content in the free air above but low vapor pressure at the mountain top and an inversion just above it to prevent normal convection, the factor in question would become abnormally large.

(*b*) *Diurnal variation in relative distribution of water vapor with height.*—As is well known the diurnal march of vapor pressure at the surface generally shows a regular periodic variation. Inland regions in summer show two maxima and two minima, occurring at about 6 to 9 a. m. and 8 to 9 p. m., for the former, and 3 to 4 p. m. and 3 to 4 a. m., for the latter (**12**[b]). In general, the oceans in summer and winter and most inland regions in winter show but one maximum and one minimum, similar to the diurnal march of temperature, the maximum occurring during the afternoon and the minimum during the early morning. Coastal stations show variations between the extremes outlined above, but resemble the oceans most closely.

The causes of this diurnal march of absolute humidity at the surface are substantially as follows. In *summer*, at *inland* stations, the ground at dawn is greatly cooled due to the nocturnal radiation, especially so if the night has been clear. The subsidence of the air during the night due to this cooling and to the relative absence of convection carries much moisture down to the ground layers from the atmosphere. These two processes conduce to the process of condensation near the ground, and the formation of dew, especially if vegetation is present. Hence the low temperatures near the ground cause the space to have a smaller capacity for water vapor and also cause the removal under proper circumstances of much of the water vapor by condensation, producing a minimum of vapor pressure and absolute humidity near the ground just before dawn. This is the so-called secondary minimum.

As the sun rises, it warms the ground and evaporates much moisture. The lapse rates at first are insufficient to cause much instability hence the vapor pressure rises to the primary maximum occurring between 6 and 9 a. m. The "nocturnal inversion" frequently found not far above ground also aids by acting as a sort of ceiling to prevent the moisture from diffusing rapidly aloft. When the sun gets higher, the lapse rates increase near the ground, and often the "nocturnal inversion" disappears or rises higher in a less marked state. Thus convection becomes active, carrying much water vapor away from the ground layers. By the time the afternoon maximum of temperature has been reached, the supply of surface ground water has been greatly depleted and the rate of evaporation from the ground has become less than the rate at which the ascending air currents and eddies carry the moisture aloft. Hence we have the primary minimum of vapor pressure (and absolute humidity) at the surface occurring about mid-afternoon in the summer at inland stations. The evening (secondary) maximum occurs as a result of the rapid subsidence of air at dusk or shortly thereafter when convection has greatly diminished, and also as a result of the comparatively small decline in temperature near the ground.

Tropical stations in general present the characteristics described above all the year round.

Over the ocean in summer and winter the sun does not warm the water very rapidly and the diurnal amplitude of temperature is small, hence no rapid increase of evaporation can take place immediately after dawn and the morning maximum is absent. As the altitude of the sun increases, the rate of evaporation increases. Since an indefinitely large supply of water is available, and for other less important reasons not presented, the evaporation can provide more water vapor than is removed. Hence we have an afternoon maximum. The evening maximum is also absent, here largely because the great ocean mass and slow change of water temperature prevent marked changes in surface evaporation, and decrease the tendency for sudden subsidence. The minimum occurring before dawn results from nocturnal cooling of the surface water and lower strata of air. Coastal and island stations are greatly influenced by the ocean and in general show the same type of diurnal march of surface vapor pressure.

At inland stations in winter the diurnal amplitude of temperature is usually comparatively small; and generally a considerable amount of surface ground water is available, either in the form of a snow-cover or ground frost. Also, inversions are quite prevalent over many temperate stations in winter (see Table 14), persisting in some cases throughout the day. These factors, and others, combined with the low altitude of the sun conduce to a slow and often small increase of vapor pressure at the surface from dawn to the afternoon maximum. Convection being relatively weak, the surface supply is little depleted thereby. The evening subsidence is comparatively less marked than in summer and ground temperatures are quite low, hence the evening maximum does not occur. The early morning minimum is caused by the same processes as were previously described.

With regard to mountains, the diurnal variation is similar to that of the free air some distance above the ground. Thus, convection carries moisture up the mountain sides from the valleys in the afternoon at about the time the sun is most effective in producing evaporation from the ground water and vegetation on the mountain slopes. Hence the maximum occurs in the afternoon, and the minimum before dawn when radiation has brought about considerable cooling and much of the moisture has been carried down by subsidence.

On low hills it is possible for the valley effect to preponderate over the free-air effect and the diurnal variation of surface vapor pressure thereon to resemble somewhat that of the valley.

Similarly the vapor pressure in the free air has a periodic diurnal variation. The data presented by Hann (loc. cit. p. 253) for the diurnal march of vapor pressure on mountain tops shows that for moderate heights (2,700–3,700 m.) there is a maximum occurring between 1 and 5 p. m. in the afternoon and a minimum occurring in the early morning from 2 to 6 a. m. With regard to the diurnal variation of absolute humidity over Mount Weather, 526 m. above sea-level and 374 m. above the valley floor (**16**), Blair (**17**) has stated that—

> With the exception of the surface and 1-kilometer levels in the summer half of the year and the 2.5 and 3 kilometer levels in the winter half of the year, the maximum moisture content of the air is found shortly after noon and the minimum shortly after midnight at all levels (526–3,000 m.) and in all times of the year. At the four levels mentioned the maximum moisture content is found just before noon.

An examination of the curves of the diurnal variation of absolute humidity over this place shows that a close approximation to the mean value for the day prevails between the hours 7 to 10 a. m., i. e., the time of day represented by the data given in Tables 2, 3, and therefore most probably also Table 7. This is also borne out by Süring's data (**2**, p. 162) from balloons and Hann's data from mountain stations.

It is evident from the foregoing that for a low-lying station in summer if the total amount of water vapor in a column of air of given cross-section is greater in the early afternoon than in the period 7 to 10 a. m., and also the surface vapor pressure is less in the early afternoon than in the morning, then the factor F_s^∞ applicable to the afternoon should be greater than that for the morning. In winter, since the surface maximum of vapor pressure falls in the afternoon, the opposite of this may be true, particularly where a snow cover exists. Likewise for mountain stations, either of these conditions may obtain, depending on the height, since if the mountain is sufficiently high the maximum surface vapor pressure occurs in the afternoon. This then introduces another source of error in the use of the factors given, indicating that both diurnal and altitudinal corrections are necessary where they are to be used for times and heights other than those for which the data apply.

To obtain an approximate quantitative idea of the error arising from diurnal variations, the data presented by Blair (loc. cit.), for Mount Weather, Va., showing the diurnal variation of temperature and absolute humidity for the surface (526 m.), and the levels for every 500 m. interval from 1,000 m. to 3,000 m. inclusive, all above sea level, were used to compute the respective values of F_{526}^{3000} for two seasons and two times of day each. The seasons given were summer, represented by the 6-month period April–September inclusive, and winter, represented by the period October–March inclusive. Table 9 shows the results of the computations.

TABLE 9.—*Diurnal variation of F., Mount Weather*

Summer		Winter	
Time of day	F_{526}^{3000}	Time of day	F_{526}^{3000}
8:30 a. m.	1,251	8:30 a. m.	1,434
4:00 p. m.	1,389	3:00 p. m.	1,392

The earlier times of day used are closely representative of the average time of flights upon which the data given herein are based. The later times are approximately the times of maximum water-vapor content of the air column in question. A comparison of the values shows that in summer the value F_{526}^{3000} is 11 per cent greater at the afternoon maximum, and in winter 3 per cent less than at the 8:30 a. m., average condition. Since the vapor pressure at Mount Weather is tempered somewhat by the free air overlying the adjacent valleys, it is to be expected that a valley station would find the corresponding afternoon value more than 11 per cent greater in summer and not quite 3 per cent less in winter.

As is to be expected, the diurnal variation of absolute humidity is relatively small at 3,000 m. and probably is vanishingly small at 6,000 m. On this account the actual diurnal variation in F_s^∞ during summer at a valley station may be expected to be slightly smaller than the above value or of the same order of magnitude. This is also true for winter but to a much greater extent.

In the case of stations situated on fairly high mountains, the vapor content of the air column may average only slightly more in the afternoon than in the early morning. However, increased vapor content in the free air, increased evaporation from the mountain sides with increased insolation, and forced convection of humid air up the slopes during the afternoon cause the surface vapor pressures in such cases to be disproportionately high compared to the free air some distance away. It is thus obvious that the F_s^∞ for the afternoon under such circumstances averages lower than for the early morning (**11**). Since this is contrary to what obtains at valley stations in the summer, levels must exist at which the variations in the factor are comparatively negligible on the whole, particularly on mountain slopes. In this connection we may note that the mean value of F_{1730}^∞ found by Fowle for the late morning observations at Mount Wilson was but 73 per cent of the early morning value. These values were based on days during the summers of 1910 and 1911 when spectrobolometric observations were made (**11**).

In conclusion of this topic it may be said in the absence of other data that the factors F_s^h given herein are unsafe for use at mountain stations. For valley or plain stations at heights comparable to those of the eight base stations used, corrections for diurnal variation and height are necessary. It may be suggested that during the warm part of the year a diurnal correction be used, assuming tentatively say a 12 per cent increase in F_s^∞ at the afternoon maximum (3 to 4 p. m.), over the 8:30 a. m. average value, using proportionate amounts for intermediate times, if values for these times be desired. In the spring and autumn when convection is weak a smaller value than the above should be assumed, perhaps 5 per cent. In winter, the diurnal correction may be neglected or be assumed to have a small negative value (say 2 per per cent at the afternoon maximum), especially when the ground is rather moist. Southern stations in summer may have slightly larger values than the above.

Stations situated on slightly elevated terrain should use slightly smaller values than those given above.

(*c*) *Transient variations with weather types.*—The laws governing the genesis of the macroscopic meteorological systems of the atmosphere, the cyclone and anticyclone, in some manner not entirely clear, condition the relationships between the various meteorological factors to be observed in their vertical cross-sections, so as to bring about wide divergencies. This is particularly true of the relative vapor content found from level to level in a vertical section of the lower troposphere. To emphasize this point we reproduce in Figures 20–22, inclusive, curves of the function f_h as computed from mean vapor pressures and temperatures observed in different quadrants of well-pronounced HIGHS and LOWS at several stations. Sets of curves were chosen which showed the widest divergence in this respect among all the curves available from Samuels's study of aerological observations made in well-pronounced HIGHS and LOWS (**18**).

The curves representing the average for all types of conditions are also shown by way of comparison. It is noteworthy that the curves for summer do not show such marked differences as found for the winter curves. Table 10 shows the comparative values of the integrals F_s^h for the curves given in figures 20–22, and also mean surface vapor pressures for each case.

TABLE 10.—*Examples of widely divergent values of F_s^h for special weather types in winter*

Station	Well-pronounced LOWS			Average of all types		Well-pronounced HIGHS			h
	Quadrant	$\bar{e}_s$	F_s^h	$\bar{e}_s$	F_s^h	Quadrant	$\bar{e}_s$	F_s^h	
		mb.		*mb.*			*mb.*		*m.*
Drexel, Nebr. (s=396 m.)	4	6.00	1,640	3.66	2,210	3	2.64	3,060	4,000
Ellendale, N. Dak. (s=444 m.)	3	3.57	1,850	2.56	2,170	2	1.09	4,580	3,500
Royal Center, Ind. (s=225 m.)	1	6.13	2,490	4.32	1,680	3	3.85	1,220	3,000

It should be noted that the values under LOWS and HIGHS in the table have less weight than the values in the central columns, mainly since they are based on fewer observations than the latter.

We may conclude from these values, however, that the transient variations of F^h are likely to be of such magnitude that serious errors may result if one attempts to compute the amount of water vapor in an air column at a particular moment from the average values of F_s^h given. This is most probably more true in winter than in summer. The use of average values may be safe for computing the average vapor content of the air column over a period of perhaps a season where a normal sequence of weather changes has occurred. In such cases the mean surface vapor pressure for the period must be used.

(*d*) *Errors due to sampling.*—As with every set of statistical variables where relatively few samples are taken for study, some uncertainty in the data must exist. Since the monthly means upon which the results are based were not in convenient shape to compute the probable errors, this index of the reliability of the means is not available. In all cases with the exception of the airplane flights at Washington, D. C., the means of ascent and descent were used. This method takes the diurnal variation into account and renders the final results more reliable. As stated before, where the observations are quite numerous as may be seen is the case for the lower levels at most of the stations (see Table 2), the results may be considered fairly reliable as averages.

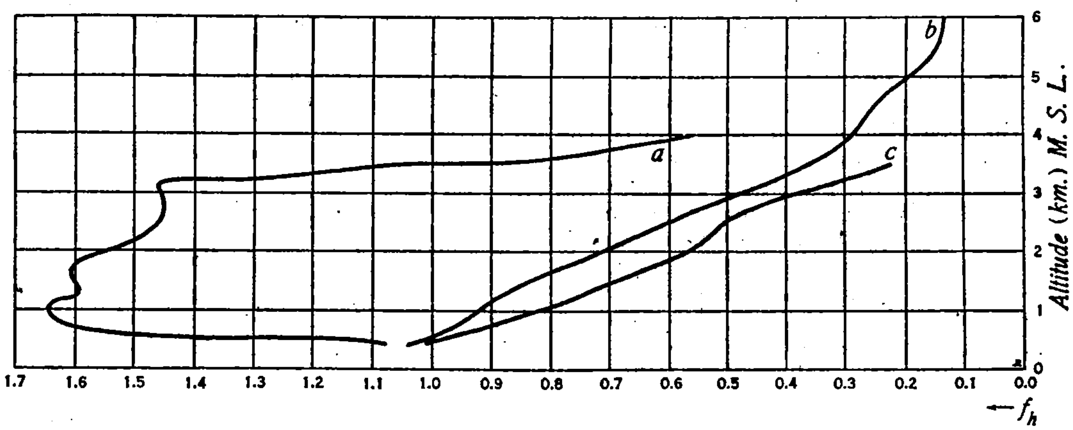

FIGURE 20.—Ellendale, N. Dak. Winter. f_h plotted against height. Curve a represents 2d quadrant of HIGHS; curve b, average of all sorts of conditions for the entire season; curve c, 3d quadrant of LOWS

Several sources of error due to sampling creep in however. Thus for example, since a certain minimum surface wind velocity is necessary before kites may be launched, it is to be expected that calm days are not well represented in the results. This is most likely to be true for the summer and autumn data and most pronounced in southern stations where more days of calm prevail during those seasons. This same effect causes the results to be less reliable in the upper levels for these seasons. Likewise, days of very strong winds are not fairly represented in the data. This is likely to be most effective at northern stations during winter and early spring. The former source of error is not present in the case of airplane observations.

In addition to the above, days of heavy or moderate rain or snow are not represented in the data. Days of low overcast sky are also lacking from the airplane data, as are data for the interior of deep banks of clouds. Kite observations on the contrary frequently provide such results.

The fact that the highest kite and airplane observations were probably made on relatively dry days brings to bear a systematic error of uncertain magnitude in the values for the higher levels.

Since nothing definite may be said regarding the magnitude of the errors arising from the above sources, it is necessary to leave the matter standing. It is felt however, in the case of kite stations where observations are numerous that the errors, if important at all, are only worth considering in the southern stations during the summer and perhaps the autumn seasons. The airplane data for Washington, D. C., are probably more nearly representative on the whole of fair and partly cloudy conditions.

(*e*) *Errors in observed values.*—As is well known, the hair hygrometers such as are used in kite, airplane and sounding balloon meteorographs are somewhat erratic in their behavior and are often subject to considerable errors. The most important source of error is probably that due

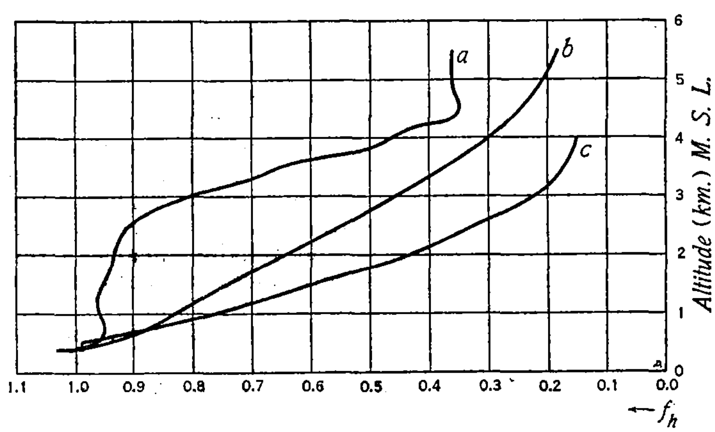

FIGURE 21.—Drexel, Nebr. Winter. f_h plotted against height. Curve a represents 3d quadrant of HIGHS; curve b, average of all sorts of conditions for the entire season; curve c, 4th quadrant of LOWS

to the effect of the lag or inertia of the hygrometric element. The investigation of Kleinschmidt (4), on this phase of the question brought him to the conclusion that the factors which cause the greatest increase in the sluggishness of the element are: (*a*) Low temperature, (*b*) low humidity, especially when the difference between the actual and recorded humidities is small, (*c*) rapid rate of change of humidity with time as regards the instrument, (*d*) large number of hairs used in the element, (*e*) poor or unequal ventilation, (*f*) poor quality or treatment of hairs. By far the most important factor of these seems to be temperature, for it is stated (loc. cit.), that—

> The temperature effect on the lag is small between +20° and +5° C.; from that temperature however, it increases rapidly, becoming infinitely great at −40° C., and almost completely reducing to nought the ability of hair to react to water vapor.

Despite objections recently raised to Kleinschmidt's methods (**19**), there is not much doubt that below −40° C., the hairs used, function more as a thermal element than a hygrometric element. This conclusion is amply supported by the indications of sounding-balloon observations.

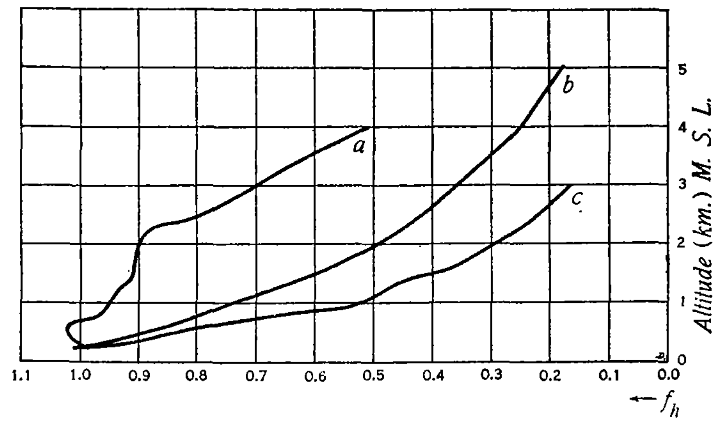

FIGURE 22.—Royal Center, Ind. Winter. f_h plotted against height. Curve a represents 1st quadrant of LOWS; curve b, average of all sorts of conditions for the entire season; curve c, 3d quadrant of HIGHS

It should be remembered, however, that meteorological kites rise much more slowly on the average than either airplanes or sounding balloons and hence the hygrometric elements have a much longer time available in which to respond to the humidity of the air than is the case for the latter methods of observation.

The lag of the temperature element is quite small in the kite instruments used (**20**), hence mean vapor pressures based on kite observations probably are more reliable than any others extant, except possibly those obtained from manned balloons and carefully conducted airplane observations. Even here, however, they must be sufficiently numerous to form a satisfactory basis for reliable results. This feature of the problem causes the values for Leesburg, Ga., to be of much less weight than the remainder of the values, since the observations taken at that place were relatively few. Likewise the values for high levels, especially in winter and early spring, are probably much less reliable owing to the temperature effect.

(*f*) *Errors due to methods of computing results.*—As stated in a previous section (III), the method of differences has been employed in computing mean monthly vapor pressures and temperatures. Since vapor pressure does not vary linearly with height, it is problematical whether that method is the proper one to use in obtaining means of that variable.

A consideration of the effects of the use of this method leads to the conclusion that if in the long run the higher observations are made on relatively dry days, as is quite likely, the computed mean vapor pressures for the higher levels will tend in the long run to be *higher* than the true means. The proper method to use is one based on the indications of the Theory of Probability and Errors considering the nature of the law of variation of vapor pressure with height. Thus far no satisfactory method that does not involve a prohibitive amount of work has been suggested, as far as known.

We thus have from these sources, errors due both to the method of computing and to a systematic limitation on sampling of data under all conditions.

In addition, another possible source of error may lie in the fact that the absolute humidity computed from the arithmetical mean of the daily observed absolute humidities for any given period may differ from the absolute humidity computed from the mean vapor pressure and temperature respectively for the same period (**21**). An examination of data for a number of months taken at random appears to show that for periods as long as a season the error in most cases falls well within the degree of accuracy of the individual observations and is of the order 1 to 2 per cent.

Probably the greatest source of error, if it is desired to use the function f_h (or F_s^h), to compute the absolute humidity at any given height from the mean surface vapor pressure for the season, results from the wide deviations of the daily ratios $\left(\frac{e_h}{e_s}\right)$ from the "mean" ratio$\left(\frac{\bar{e}_h}{\bar{e}_s}\right)$.

It is obvious from the nature of the function in question that the necessary condition that f_h give statistically correct results is that—

$$f_h=\frac{1}{n}\,\frac{273}{\frac{1}{n}\sum_{i=1}^{n}e_{s_i}}\cdot\sum_{i=1}^{n}\frac{e_{h_i}}{T_{h_i}} \tag{31}$$

where T_{h_i} = absolute temperature at height h, for the i-th observation and n = number of observations, the other symbols being as defined before, and the observations equally spaced in time. Tests on data for a number of seasons taken at random showed that percentage errors in the lower kilometer are usually quite small but are likely to increase above that height. For periods as short as a month the errors from this source may be very large for heights 2,000 m. above sea level and higher. In one case, viz, for March, 1926, at Ellendale, N. Dak., this error at the 3,000 m. level was 24 per cent of the average of the 16 observations available. When data for a full season are examined and compared, the percentage error resulting from the use of f_h for a height of say 3,000 m is found usually to fall within 7 per cent. Probably the errors would be quite serious for heights above 4,000 m.

Another source of errors falling within this category (**f**) is that arising from the use of hair hygrometer humidity readings and tables of saturation vapor pressures to compute current vapor pressures. In this method, the saturation vapor pressure corresponding to the observed current temperature is obtained from tables and multiplied by the relative humidity reading to give the current vapor pressure. For temperatures below 0° C., the tables used are those for the pressure of aqueous vapor over ice, while for temperatures above 0° C., the tables used are for vapor pressures over water. This arbitrary rule even though justified by expediency may be improper for use in the free air since for example water droplets may exist in the free air at temperatures far below the freezing point (**22, 23**). Thus, the hair hygrometer, calibrated at room temperature, when taken into the free air, yields a "number" which we call the "relative humidity." The definition of the latter term depends upon the form and kind of surface, whether water or ice, to the saturation vapor pressure of which at the given temperature we refer the actual vapor pressure to obtain the relative humidity. If for every case where temperatures below 0° C. are observed, we use the saturation vapor pressure over a flat surface of ice as the standard, and if liquid water is present in the atmosphere under the given temperature, then it is obvious that the "number" taken as the "relative humidity" may give erroneous results.

The following figures are illustrative. From the Smithsonian Physical Tables, seventh revised edition, we find for −16° C.,

1.315 mm. Hg. = saturation vapor pressure over water.

1.142 mm. Hg. = saturation vapor pressure over ice.

For 100 per cent relative humidity at this temperature with respect to water, the relative humidity with respect to ice is

$$\frac{1.315}{1.142}\times 100 \text{ per cent} = 115.1 \text{ per cent.}$$

For −30° C., Robitzsch (**24**) finds the corresponding value to be 133.2 per cent. It is obvious from these figures that if the "number" obtained from the hair hygrometer represents the relative humidity with respect to water, say at −16° C., then this "number" must be multiplied by 1.15 to obtain the relative humidity with respect to ice. In other words the vapor pressure computed as in the past from the tables for the saturated vapor pressures over ice will be 15 per cent too small under these circumstances.

The above considerations are strictly applicable only for pure substances. However, water droplets in the free air are nearly spherical and contain hygroscopic nucleii which lower the vapor pressure. The importance of these nucleii in the mechanism of undercooling of water droplets has been much emphasized by Köhler (**22**). In addition, undercooled water particles of such smallness that they are invisible must exist in the atmosphere under certain circumstances and probably are quite prevalent in the vicinity of clouds [(**22**) (b) pp. 13–15, (**25**)]. These conditions complicate the problem to such an extent that considerable uncertainty exists as to what the "number" rendered by the hair hygrometer means physically. Therefore, little can be said on this point that can be considered conclusive; however, the shadow of doubt is thrown upon vapor pressures and values computed therefrom when obtained from hygrometer readings at temperatures below 0° C. This entire subject is greatly in need of intensive and critical investigations to provide practical and reliable means of obtaining accurate vapor pressure measurements in the free air.

(*g*). *Errors due to the use of equation 25 (for reduction of given data for use at neighboring stations).*—Where equation 25 is used to compute the mean vapor content in the air column over a section other than one for which data is given herein, the largest error likely to result is that due to geographical interpolation. Thus, values S computed from the three nearest "datum stations" may show a considerable difference. This necessitates that the values be weighted according to climatological and physiographic considerations and also according to distance and direction of each station from the others. The percentage error arising from this source is obviously variable and depends somewhat upon the intimacy of the person using the formula with the nature of the region with which he is concerned. It may be mentioned here that a defect to be found in *all* formulas of this sort is that they can not take into account local or geographical variations.

The data given herein are therefore most advantageous for use in central and eastern United States since some cognizance may then be taken of these factors.

Other errors associated with the use of this equation depend on the differences between the absolute humidities existing in the free air over the "datum station" at given heights above sea level and those existing at the same heights above sea level over other stations. Several computations have been made to ascertain the magnitude of this error, using certain assumptions based on observational data. The percentage errors in these cases were found to be less than 3 per cent where the upper base of the column was as much as 5,000 m. and where $x=750$ to 1,500 m. above sea level.

Uncertainty regarding the most applicable value of the constant in Hann's equation, 8, likewise introduces the possibility of a further error. However, the value used (6,300) is considered to be the best value extant for this purpose, firstly, because it is based on mountain observations, and secondly, because it agrees well with values obtained from the data for the lowest kilometer over the stations used herein.

(h) *Miscellaneous errors.*—Among these may be mentioned (*a*) errors in the determination of e_s or e_x, (*b*) errors due to the effect of hygroscopic particles in the atmosphere, (*c*) error in the constant K depending on variations in the relative density of atmospheric water vapor to pure dry air.

As is well known, serious psychrometric errors may arise during the winter when subfreezing temperatures prevail, hence the surface vapor pressures must be determined as accurately as possible to reduce the error to a minimum.

Regarding hygroscopic particles, it may be said that very little is known as to their effect on hair hygrometers and errors resulting therefrom. In general it may be seen that hygroscopic nuclei permit of a larger moisture content in the air than would appear possible from theoretical considerations which disregard their presence (**22**). This brings in an error whose magnitude it is difficult to gage under present circumstances. As was mentioned before, this is one of the problems for the future.

The influence of electrical charges and ions may be of material importance in this regard.

Possible errors in the constant $K(=1.060$ for e_s in mm.) may be dismissed as of small importance compared to the other errors since they probably amount to but a few tenths of a per cent within the range of temperatures thus far observed in the troposphere (**26**).

It is necessary to emphasize here that the present study does not take into account the water which is present in the atmosphere in the liquid form. Although the mass of water vapor per cubic meter of cloud has been found always to exceed the mass of liquid water present in the same volume, the latter may become as great as 5 grams per cubic meter in the heaviest clouds as has been shown by the independent investigations of Conrad, Wagner, and Köhler (**27**).

VI. COMPARATIVE STUDY OF THE DATA

1. *The function* $f_h=\left\{\dfrac{\left(\dfrac{e_h}{e_s}\right)}{1+\alpha t_h}\right\}$.

(*a*) *Seasonal variation.*—A study of the values of f_h given in Table 2 shows that on the average the values are greatest in winter and least in summer, and usually for heights greater than several hundred meters above ground the autumn values are greater than the spring values. Also, it may be seen that the values for summer for certain levels (usually above 1.5 km.) are greater than the values for spring. In southern stations where this is most pronounced, the summer values even exceed the autumn values for certain levels.

The interpretation of the statement that f_h for a given level is greater for one season than for another is that the absolute humidity at that level is greater on a day during the first season than on one during the second season where the vapor pressure at the surface is the same in both cases.

The contrasts between the various seasonal values depend partly upon the temperature differences existing and partly upon actual changes in relative vertical distribution of water vapor. It is evident from the gas laws that for a given vapor pressure the vapor content of a

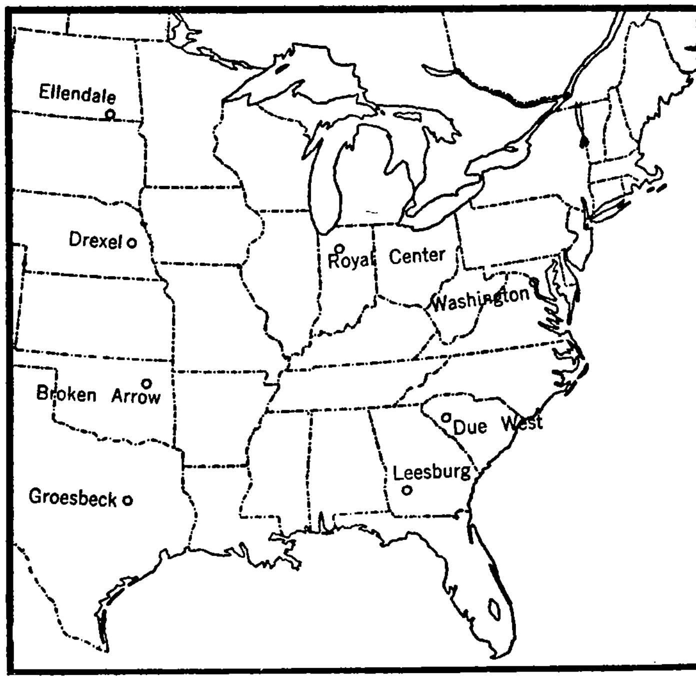

FIGURE 23.—Geographical locations of the eight stations used herein

given volume is greater at low temperature than at high temperature.

If the ratios $\dfrac{f_h}{f_s}$ be formed from the data given in Table 2, it will be seen that the ratios are greater in winter than in summer at the four stations Drexel, Ellendale, Groesbeck (note below), and Washington, D. C. The reverse is true for certain intervals of height at the other stations. The intervals where $\left(\dfrac{f_h}{f_s}\right)_{\text{summer}}>\left(\dfrac{f_h}{f_s}\right)_{\text{winter}}$ are:

Broken Arrow, from 250–500 m. to 2,000–2,500 m.
Due West, from 2,000–2,500 m. to beyond 4,000 m.
Groesbeck, from surface–250 m. to 500–750 m.
Leesburg, from surface–250 m. to beyond 4,000 m.
Royal Center, from surface–250 m. to 1,500–2,000 m.

It will be noted that Groesbeck shows this effect only slightly and that the winter ratios are greatest at stations where in general the winter inversions are most pronounced (see figs. 20–22, and also ref. (**18**)). Referring back to Section V, 2 (*a*), p. 461, a number of causes operating to produce this relationship in inversions have been discussed.

It may be added here that when convection and turbulence are most active, i. e., when lapse rates are near the adiabatic, the water vapor distribution naturally shows a more nearly uniform manner of decrease with height than when inversions are present. In the latter case the tendency is for the water vapor to stratify within or just below the inversion and to show a sharp decrease just above it. We should therefore consider these factors as among the most dominant in producing the downward march of the water content of the upper troposphere from summer to winter and its concentration in the lower few kilometers in the latter season, particularly in regions farthest removed from the Equator.

(*b*) *Geographical variation.*—Since the stations used herein are not of equal elevation and since the periods of observations upon which the present data are based are not identical, nor, of equal length, nor of very great duration, comparisons between the several stations must be taken with some reservations. Such comparisons with respect to vertical position should, strictly speaking, be comparisons between data for equi-geopotential surfaces (**28**), or possibly even surfaces of equal gravity potential above ground. Unfortunately, reduction of the data to such surfaces involves a large amount of additional labor. Such reductions are of course more important for high levels and for extensive latitudinal differences, but since the reliability of the data scarcely justified this refinement they were not undertaken.

The latitudinal variation of f_h may be seen by a comparison of the data for Ellendale, Drexel, Broken Arrow, and Groesbeck in order. The function shows a progressive decrease from north towards south at all levels in the lower 3–4 or so km. over these stations. Above these heights the relationship is not so consistent but signs of a reversal are evidenced. Comparing the data for Washington, D. C., and Due West, it would appear that f_h for the former is less at all levels during the summer and autumn, while during the other two seasons it is less only in the lower few kilometers but is greater above that height. Likewise, comparing Due West and Leesburg (data least reliable), it would appear that the data for Due West are greater at all levels in autumn and winter. During spring and summer however, f_h for the former is only greater from the surface to 2.5–3.0 km., the opposite being true above these heights.

Something regarding the longitudinal variation may be seen by comparing Drexel with Royal Center, Royal Center with Washington, Broken Arrow with Due West, and Groesbeck with Leesburg. Values of f_h at Drexel are found to be greater than those at Royal Center at all levels and all seasons. The relationship between Royal Center and Washington values is more complex. Speaking in general, the values at the former station are greater in the lower layers (surface to 750–2,500 m. depending on season), then the reverse is true for a thousand or more meters, and finally there is some evidence that at greater heights the Royal Center values are again greater.

Considering Broken Arrow and Due West, during summer and autumn for heights beyond the lower half kilometer or so, the Due West values of f_h appear greater than the Broken Arrow values. During the other two seasons, this is only true to heights between 2.5–3.0 km., a reversal of the relationship appearing above these limits. Groesbeck values show themselves to be greater than the Leesburg values in the lower kilometer or so (roughly speaking) but less above these heights in all seasons except autumn which has a more complex connection.

The interpretation of such relationships as are described above has already been given in the preceding section (*a*). Attention is invited to the fact that the values of f_h particularly for the lower levels appear to be smaller for stations near bodies of water than for inland stations considerably removed therefrom. This relationship is most pronounced in the North. This circumstance may be largely due to other local conditions [1] and hence must be investigated further to obtain verification or disproof of such a general conclusion.

2. *The average absolute humidity aloft.*—$\overline{W}_h = K\bar{e}_s f_h$, *g./m.*[3]

(*a*) *Seasonal variation.*—Table 11 has been computed according to the above equation from data given in Table 2.

TABLE 11.—*Geographical and seasonal variation of absolute humidity* (*g./m.*^{3})

Height above sea level (m.)	Ellendale (444 m.)	Drexel (396 m.)	Broken Arrow (233 m.)	Groesbeck (141 m.)	Royal Center (225 m.)	Washington (7 m.)	Due West (217 m.)	Leesburg (85 m.)
SPRING								
Surface	4.90	6.32	9.11	11.51	6.77	7.44	9.10	11.10
250	----	----	9.03	11.06	6.67	6.74	8.96	10.26
500	4.77	6.01	8.02	9.99	5.81	6.02	8.02	9.33
750	4.22	5.35	7.23	9.05	5.23	5.39	7.32	8.59
1,000	3.84	4.86	6.61	8.03	4.75	4.95	6.75	7.93
1,250	3.52	4.41	5.94	6.97	4.29	4.58	6.19	7.27
1,500	3.22	3.99	5.28	5.94	3.89	4.27	5.59	6.54
2,000	2.65	3.25	4.16	4.40	3.19	3.53	4.40	4.84
2,500	2.16	2.69	3.30	3.48	2.47	2.82	3.37	4.14
3,000	1.72	2.24	2.68	2.86	1.95	2.21	2.56	3.36
3,500	1.37	1.83	2.21	2.36	1.57	1.80	1.97	3.04
4,000	1.07	1.49	1.77	1.99	1.27	1.36	1.58	2.78
SUMMER								
Surface	11.74	13.84	16.84	18.26	13.68	15.84	16.17	17.89
250	----	----	16.70	17.70	13.51	14.46	15.95	16.71
500	11.40	13.09	14.94	16.19	11.99	12.92	14.39	15.48
750	10.05	11.57	13.51	14.29	11.02	11.66	13.23	14.67
1,000	9.09	10.53	12.34	12.43	10.20	10.60	12.25	13.55
1,250	8.25	9.62	11.21	11.03	9.32	9.69	11.26	12.27
1,500	7.44	8.71	10.13	9.86	8.37	8.96	10.25	10.92
2,000	6.05	7.12	8.16	7.95	6.51	7.54	8.39	8.84
2,500	4.99	5.77	6.46	6.49	4.84	5.95	6.85	7.27
3,000	4.04	4.66	5.18	5.33	3.68	4.51	5.56	6.47
3,500	3.32	3.76	4.15	4.39	2.79	3.45	4.60	5.63
4,000	2.74	2.99	3.30	3.62	2.16	2.52	3.73	5.14
AUTUMN								
Surface	5.83	7.27	10.19	12.80	8.51	10.18	10.34	13.47
250	----	----	10.12	12.36	8.42	9.29	10.18	12.73
500	5.72	6.97	9.14	11.30	7.55	8.49	9.23	11.82
750	5.22	6.31	8.31	10.31	6.91	7.79	8.48	10.90
1,000	4.77	5.78	7.64	9.23	6.27	7.23	7.88	9.92
1,250	4.33	5.29	6.97	8.19	5.65	6.68	7.21	9.04
1,500	3.93	4.84	6.25	7.29	5.05	6.12	6.55	8.00
2,000	3.26	4.04	4.84	5.61	3.99	4.93	5.20	5.95
2,500	2.73	3.36	3.71	4.32	3.14	3.81	4.14	4.36
3,000	2.27	2.77	2.88	3.31	2.52	2.83	3.41	3.34
3,500	1.85	2.25	2.28	2.64	2.02	2.08	2.88	2.62
4,000	1.51	1.87	1.66	2.06	1.46	1.42	2.45	2.33
WINTER								
Surface	2.11	2.96	4.75	7.13	3.47	3.85	5.09	7.60
250	----	----	4.71	6.84	3.41	3.59	5.92	7.09
500	2.07	2.82	4.21	6.24	3.03	3.30	5.39	6.44
750	1.95	2.59	3.78	5.72	2.77	3.09	5.04	5.97
1,000	1.88	2.44	3.42	5.12	2.51	2.86	4.70	5.45
1,250	1.81	2.32	3.07	4.59	2.27	2.65	4.33	4.95
1,500	1.70	2.17	2.76	4.05	2.06	2.45	3.93	4.46
2,000	1.45	1.87	2.23	3.17	1.69	2.09	3.19	3.43
2,500	1.22	1.59	1.85	2.57	1.44	1.76	2.51	2.82
3,000	0.97	1.34	1.56	2.09	1.24	1.42	1.99	2.32
3,500	0.74	1.10	1.34	1.70	1.04	1.14	1.58	1.68
4,000	0.59	0.87	1.14	1.45	0.86	0.95	1.30	1.23

Comparison of the data by seasons shows that there is a progressive increase in absolute humidity from winter to summer and that the autumn values exceed the spring values at all the stations and for almost all the levels given. The levels 4,000 m. at Broken Arrow and 3,000–4,000 m. at Leesburg stand as exceptions (note data for latter station not very reliable).

(*b*) *Geographical variation.*—Figure 23 indicates the geographical location of the eight stations used. Com-

[1] See discussion on p. 455, Section IV, 2, regarding low temperature in the free air along the Atlantic Coast.

parisons of the stations presented in the first four and last three columns of Table 11 indicate the progressive increase of absolute humidity on going from north to south at all levels given. Broken Arrow, 4,000 m., autumn; Leesburg, 3,000–4,000 m., autumn; and Leesburg, 4,000 m., winter, stand as exceptions. The Leesburg values being based on few observations, are not very reliable and hence these exceptions are to be taken with reservations.

Comparing Drexel and Royal Center we find the values for the former to exceed those for the latter at all levels above 500 m. in spring, and at all levels in summer. During autumn the Drexel absolute humidities are less than the Royal Center absolute humidities from the surface to between 1,500–2,000 m. Above that height the Drexel values are greater. In winter the same relationship exists, only the height at which the reversal takes place lies between 1,000–1,250 m.

The relationships last presented appear anomalous at first sight, for one would be inclined to think that the proximity of Royal Center to Lake Michigan would render it more moist aloft than an inland station far removed from the lake and almost equidistant from the Gulf of Mexico. However, they may be traced back to the pressure gradients which normally exist over continental United States, and to the resulting air flow from different origins. Referring to Gregg's **(29, 6)** Aerological Survey of the United States (Mo. Wea. Rev. Supp. 26, pp. 55–56 and Supp. 20, pp. 39 and 45) it will be seen that in summer and spring the normal pressure gradients cause the resultant winds over Drexel to have a considerable southerly component while the resultant winds at Royal Center are more from the west and west-northwest. This brings about a greater transport of moist gulf air to Drexel than to Royal Center, and the latter must get a larger proportion of the relatively dryer polar air **(30)**. In winter and autumn the resultant winds at Drexel have a more northerly component than those for Royal Center and the relationship is partly reversed.

Comparisons of Royal Center with Washington, Broken Arrow with Due West, and Groesbeck with Leesburg bear out remarkably well on the whole what would be expected from considerations of the resultant air flow.

These facts emphasize the importance of studying the movement of air masses more closely **(30)**, both for forecasting purposes and for the study of comparative climatology.

3. *The integral,* $F_s^h = \int_s^h f_h dh$.—

(*a*) *Seasonal variation.*—Considering the values given in Table 3, it will be noted that the winter values are the largest. In northern stations the summer values are always least for the data given. In southern stations the summer values differ little from or exceed the spring values for h generally above 2,500 m., the summer values being less for h below that approximate height. Leesburg appears to show this difference at even lesser heights. The autumn values exceed the spring values in every case where the data are relatively reliable. Leesburg above 3,000 m. may be an exception.

The interpretation of a statement that F_s^h for one season exceeds the corresponding value for another season is that on days when the surface vapor pressures are the same in both seasons, the day in the first season will have a larger total vapor content, S^h_s, in the air column from the surface to height h than will the day in the second season.

Some of the underlying causes of the differences indicated above have been previously discussed under Section VI, 1 (a).

(*b*) *Geographical variation.*—Since the values of F_h have not been reduced to a common datum surface, they are not strictly comparable. However, since it so happens that the group of stations Ellendale, Drexel, Broken Arrow, and Groesbeck have lower surface elevations above sea level in descending order respectively, some valid conclusions may be drawn from the data given. An inspection of the values for these stations indicates that in the higher levels at least, the values decrease from north to south, despite the opposing effect of decreasing surface elevation in the same direction. Hence it may safely be concluded that if the data were reduced to a common datum surface, the values, for h (the upper limit of the column) equal to say 4,000 m., would decrease from north to south. This is in accord with the general latitudinal variation found for f_h, and is most pronounced in the winter seasons as was found for the latter.

In a similar manner we note that the Drexel values exceed the Royal Center values, particularly for the higher levels.

4. *The average total vapor content of the air column.*—$\bar{S}_s^h = K\bar{e}_s F_s^h$.

(*a*) *Seasonal variation.*—As was found for the seasonal variation of absolute humidities, the values $\bar{S}_s^h$ from Table 3 may be seen to increase from winter to summer, with summer having the maximum values. The autumn vapor content exceeds the spring content in every case. The greatest contrast between summer and winter content is found in northern stations and the least in southern stations. Comparing the values for $h = 4{,}000$ m. for the various stations, it is seen that the spring content is about 0.5 the summer content in northern stations and slightly more (roughly 0.6) in southern stations. For the same upper limit, the average winter content is about 0.25 the average summer content in northern stations. The proportion increases as one goes southward, being near 0.4 at Groesbeck and Leesburg.

The relatively smaller difference between the vapor content during these two seasons in the southern stations as compared with the northern stations is partly due to the smaller contrast between winter and summer with respect to total solar radiation received at the southern stations as compared with the northern stations **(31)**. This produces a smaller amplitude of the mean free-air temperature variation between winter and summer at southern stations as compared with northern stations. This in turn influences the relative capacity of the space for water vapor and also the relative evaporation from water surfaces and the soil. The nearness of the southern stations to bodies of water also brings to bear the tempering effect of the high specific heat and slow rate of cooling of the water.

With regard to the solar radiation received, it must be remembered that even though the intensity of the solar radiation received at the top of the atmosphere per day in summer differs little between stations at latitude 30° and 40° N., the amount received at the ground is markedly greater at latitude 40°, in fact the maximum on June 21 is received at about latitude 48° N. (sea level). This is brought about by the increasing length of day and decreasing vapor content from south to north, in spite of the lower altitude of the sun at midday at northern stations **(32)**. It is thus seen that the water vapor blanket which is so effective in depleting the radiation received at the top of the atmosphere and which must increase towards the Equator as the result of the cumulative effect of more intensive heating, itself must act as a tempering agent to diminish the difference between summer and winter at southern stations. The annual march of cloudiness, the variations of which at most places in temperate latitudes can not simply be attributed to solar radiation, will also be seen to be an important factor.

Despite the greater total radiation received in spring as compared with autumn (at sea level), the total vapor content was found to be greater during the latter season. This is largely the result of the after-effect of the preceding seasons in each case respectively.

The more frequent outbreaks of the relatively dry polar air in winter and spring must also be considered an important factor governing the seasonal variation of the vapor content of the air.

(*b*) *Geographical variation.*—Considering the values given in Table 3 for Ellendale, Drexel, Broken Arrow, and Groesbeck, despite the differences in surface elevation, it may be safely said that the total vapor content, $\bar{S}_s^h$, in general increases from north to south, as is well known. This is likewise shown by the stations to the eastward, if some allowance is made for differences in elevation.

Comparing Drexel and Royal Center values, it will be seen that despite the greater elevation of the former, the summer values for Drexel exceed those for the latter station at heights above the layer between 3,500 and 4,000 m. This agrees, of course, with the marked differences in absolute humidity found between the two stations for this season. A close analysis of the spring values for these stations appears to indicate that possibly for some height above 6,000 m. the total vapor content of the column for the former may differ very little from that for the latter, this in spite of difference in elevation. This is not so likely to be true in the autumn and winter. (See tables 7 and 2.)

Broken Arrow and Due West show very small differences in $\bar{S}_s^h$ for spring, but the difference becomes more and more marked until it reaches a maximum in winter. This is probably largely due to the seasonal changes in frequency and strength of the free-air winds and their places of origin. Thus in spring the most frequent winds at 1,000 m. above surface at both stations are from the Gulf of Mexico (**29**, p. 43). The summer months show a slightly smaller frequency from the northwest quadrant, with slightly more from the southwest at Due West. The winter months on the other hand at Broken Arrow have their most frequent winds at 1,000 m. from the southwest and northwest, i. e., from relatively dry regions, while at Due West the most frequent winds in this season are from the northwest, west, and southwest. The trajectories of air flow in the lower Mississippi Valley and in the Gulf region show that much of the air reaching the southeastern seaboard of the United States in winter (as well as in summer and spring, to a lesser extent in autumn) must have its origin in the Gulf of Mexico. Hence these circumstances are to be regarded as the secondary causes of the differences to which attention was called.

Groesbeck and Leesburg show similar characteristics, if some allowance is made for differences in elevation.

As was stated before, a factor to be considered in the study of the causes of the seasonal variation of the vapor content of the air column is the question of the frequency of outbreaks of polar air. This is also important with regard to geographical-seasonal variations. Thus in winter, spring, and late autumn outbreaks of continental polar-air are more frequent than in summer, late spring and early autumn. Since Ellendale, for example, is more nearly in the path of such outbreaks than any of the other stations, it is obvious that this cause will bring about a more marked variation in $\bar{S}_s^h$ between winter and summer at this station than at any of the others. Drexel and Royal Center are also likewise affected. On the other hand, the southern stations such as Groesbeck, Due West, and Leesburg will be much less affected by this cause, since in general the polar-air will have warmed somewhat by its passage southward, and will have had an opportunity to acquire more water vapor. Furthermore, the track of winter cyclones fed by polar-air is often such as to miss entirely the southern stations.

Hence it appears that the variations noted above may largely be explained in terms of solar radiation and air trajectories, these undoubtedly being conditioned by more basic phenomena such as: The revolution of the earth in its orbit; the inclination of the earth's axis to the plane of the ecliptic; the rotation of the earth about its axis; gravity; the physical properties of water in its various forms, as well as of air and earth; the relative distribution of land and water and other physiographic features; solar radiation, quality as well as intensity, as received at the top of the atmosphere; and others.

With regard to the influence of mountain barriers on the vapor content of the air column, the station which we would expect to be most influenced among those given herein is Washington, D. C. There is some evidence that in spring, summer, and autumn the mountain barrier to the west of that station is quite instrumental in partially depleting the vapor content of the air currents which frequently in those seasons flow up the Mississippi Valley from the Gulf of Mexico and recurve eastward toward the Atlantic Ocean. The same effect is produced in winter but here quite often the supply cut off at low levels is comparatively rich in water vapor at heights above the mountain tops, due to inversions, and hence it appears likely that the contrast in vapor content between this station and one to the west of the mountains would be more striking in the former three seasons than in winter. (Compare figs. 11–13.)

(*c*) *Discussion of* $\bar{S}_s^\infty$.—Table 12, which was computed from the factors F_s^∞ given in Table 7 and the mean surface vapor pressures given in Table 2, shows the (tentative) approximate mean depth of water which would be formed if all the water vapor in the air column from the ground to the limits of the atmosphere were condensed instantaneously and deposited upon the ground. The values are given for each season and are expressed both in centimeters and inches. These values give a relative indication of the mean quantity of water vapor effective for absorbing solar radiation and earth re-radiation.

TABLE 12.—*Approximate mean depth of rain equivalent to total vapor content of air column from surface to outer atmosphere* ($\bar{S}_s^\infty$)

Station	Spring		Summer		Autumn		Winter	
	Cm.	*In.*	*Cm.*	*In.*	*Cm.*	*In.*	*Cm.*	*In.*
Broken Arrow, Okla	[1] 1.99	0.785	3.75	1.478	2.17	0.853	1.12	0.441
Drexel, Nebr	1.44	.569	3.16	1.245	1.80	.708	.80	.316
Due West, S. C	1.96	.773	3.92	1.545	2.50	.984	1.45	.570
Ellendale, N. Dak	1.10	.432	2.71	1.067	1.41	.557	.58	.229
Groesbeck, Tex	2.41	.950	4.12	1.622	2.76	1.086	1.65	.651
Leesburg, Ga	2.48	.976	4.29	1.688	2.97	1.169	1.73	.681
Washington, D. C	1.69	.665	3.49	1.372	2.23	.878	1.00	.394
Royal Center, Ind	1.45	.570	2.90	1.142	1.87	.735	.83	.329

[1] To obtain mass in kg., per column one sq. m. in cross section, multiply depth (in cm.) by 10. To obtain mass in metric tons per column one sq. km. in cross section, multiply depth (in cm.) by 10^4. To obtain mass in short tons (2,000 lbs.) per column one sq. mi. in cross section, multiply depth (in cm.) by 2.855×10^4.

It is clear from the values presented that the blanketing or "greenhouse" effect of the water vapor is more effective by far in summer than in winter. Were it not for this blanket of water vapor in summer, it is obvious that our days would be much more unbearable so far as temperature is concerned and the nights very cool. Similarly the smaller amount of water vapor in winter tends to reduce the amount of radiation absorbed by the atmosphere, hence making our winters relatively colder on this score than our summers. That is, our solar climate generates a cycle of events which tends to augment its effect in winter by its influence on terrestrial moisture, and on the contrary in summer it tends to retard and conserve its effect by its influence on the same agent. This is probably an important factor in explaining the great contrast existing in winter between polar and equatorial regions and hence the stronger gradients and more intensive circulation than in summer.

VII. SUMMARY

Tables have been introduced (2, 3, 7) for computing the average absolute humidities at various heights, and the total vapor content of m^2. columns extending from the ground to various heights above sea level, from the mean vapor pressures at the surface, for eight stations in the United States east of the Rocky Mountains.

An equation, 25, has been given to permit the use of the data given in tables 3 and 7 for other stations not too distantly located from those given and physiographically similar. The errors resulting from the methods employed have been fully discussed. It is emphasized that serious errors may result if the given factors are used to compute the required vapor contents for periods of less than a season.

Under the discussion of errors, a number of topics of more general interest have been treated. Among these may be mentioned: The vapor distribution in inversions and the mechanism involved (V, 2, a.); the diurnal variation of absolute humidity near the surface, near mountains, and in the free air (V, 2, b.); errors due to the use of hair hygrometers at low temperatures (V, 2, e.); errors in vapor pressures computed from hair hygrometer readings at temperatures below 0° C. (V, 2, f.)

The various data, viz. f_h, $\overline{W_h}$, F_s^h and $\overline{S_s^h}$ (see definitions in Sec. II), have been discussed with regard to their seasonal and geographical variations. Special emphasis has been laid on the air trajectories and solar radiation to explain some of the differences found.

A study of the relationship between average precipitation, atmospheric water vapor content, and other factors has been begun. It may be stated at this time that the mean precipitation is not directly proportionate to the mean vapor content but depends to quite an extent upon other factors also. It is hoped to publish a paper on this subject in the future.

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SOLAR RADIATION AS A METEOROLOGICAL FACTOR [1]

By HERBERT H. KIMBALL

SYNOPSIS

Variations in the earth's solar distance cause variations in the intensity of solar radiation at the outer limit of the earth's atmosphere of very nearly 3.5 per cent on each side of the mean, with the maximum early in January and the minimum early in July.

Variations in solar declination cause seasonal variations in the daily totals of solar radiation as measured at the surface of the earth, which are small at the Equator, but increase rapidly with latitude. At Habana, Cuba, latitude 23° 09′ N., the average daily amount at the time of the summer solstice is about double that at the time of the winter solstice; at Washington, D. C., latitude 38° 56′ N., the corresponding ratio is about 3.5; at Stockholm, Sweden, latitude 59° 21′ N., it is about 20, and at Sloutzk, Union of Socialist Soviet Republics, about 40.

Following explosive volcanic eruptions the great quantity of dust thrown into the atmosphere, some of it to great heights, has diminished the intensity of the direct rays of the sun as received at the earth's surface from 15 to 25 per cent for periods of several months. Such explosions, with their accompanying dust clouds, occurred in 1883, 1888–1891, 1902, and 1912, and a slight cooling of the earth as a whole seems to have followed. On the other hand, there have been no such eruptions since 1912, or during a period of nearly 20 years, and Ångström is of the opinion that on account of the small amount of dust now present in the stratosphere the temperature of the earth should be slightly higher than usual.

For solar constant values it has been claimed that periodicities of from 68 to 8 months exist, with amplitudes of from 0.005 to 0.014 calories, or about 0.3 to 0.7 per cent of the mean value. Also, that there are short-period trends in values, with an average length of five days and an average amplitude of 0.8 per cent. To these short-period trends of less than 1 per cent in magnitude, have been attributed the "Major changes in weather."

A careful study of these various variations in the intensity of solar radiation leads to the conclusion that weather changes are brought about, not by short-period trends of less than 1 per cent, but by the manyfold difference in the intensity of the solar radiation received by the earth in equatorial and polar regions. As a result great temperature differences exist between these regions. Gravity causes the heavy cold air to displace the lighter warm air at the surface, and a polar-equatorial circulation is set up, turbulent in character, especially in winter when the temperature difference is most marked. Well-defined movements of this character are to be found on the weather maps of the different countries, and examples are shown in this paper in reproductions of weather maps for the United States. It is to studies of this turbulent polar-equator movement of air that meteorologists look for improvements in weather forecasting, and it is for such studies that the meteorological work of the Jubilee International Polar Year 1932–33 is now being organized.

INTRODUCTION

Although in this paper solar radiation is to be considered from the standpoint of the meteorologist, there are certain astrophysical and astronomical facts that also must be kept in mind.

Thus, astrophysical research has shown that the sun is a hot, luminous body, perhaps gaseous throughout, with its outer layers rotating about the solar axis at different rates in different latitudes. The quality of solar radiation is about that of a black body at a temperature of 6,000° A. This may therefore be taken as the effective temperature of the sun. The temperature of its center, on account of the enormously high pressure that must there prevail, is variously estimated to be from thirty to sixty million degrees.

The sun radiates, we are told, 3.79×10^{33} ergs of energy per second, corresponding to a loss of about 4,000,000 tons of mass per second. Of this vast amount of energy the planets and their satellites intercept about 1/120,000,000, and the earth about 1/2,000,000,000, or 4.1×10^{16} gram-calories per second.

What becomes of all the solar radiant energy except that intercepted by the planets and their satellites, and how the sun maintains this enormous output of energy without apparent impairment of its resources, while interesting problems, will not be considered here. Rather, we shall confine our attention to the one 2-billionth part that is intercepted by the earth, and which is of vital interest not only because it is the source and the support of all life on the earth, but also because it is the source of weather and climate.

ANNUAL VARIATIONS IN SOLAR RADIATION INTENSITY RECEIVED BY THE EARTH

The earth is at its mean solar distance of approximately 93,000,000 miles twice each year—in 1931 on April 4 and October 5. It was nearest to the sun on January 3, and farthest from it on July 6. The ratio of the longest to the shortest distance is 1.034, and since the radiation intensity varies inversely as the square of the distance from the radiating body, other things being equal its intensity early in January should have been nearly 7 per cent higher than in early July. Therefore solar radiation received by the earth has an annual variation in intensity of about 7 per cent, and we in the Northern Hemisphere are now favored by the fact that the maximum intensity occurs during our winter.

Besides the annual variation in the earth's solar distance there is also the annual variation in the sun's apparent declination due to the inclination of the earth's axis of rotation to the plane of the ecliptic, in consequence of which the position of the sun in the heavens coincides with the plane of the terrestrial equator at the time of the equinoxes only. From March 21 to September 21 the sun is north of the terrestrial equator, or its declination is north, and during the remainder of the year it is south. During the summer months, therefore, the sun's rays strike the surface of the earth in the Northern Hemisphere at a smaller angle from the vertical, and thus have a shorter path through the atmosphere during most of the day than during the winter months; also,

[1] Presented before Section B, A. A. A. S., at a joint session with the American Meteorological Society at New Orleans, La., on December 30, 1931.